# Solution Math 301-101 Sec: 02 & 03 Quiz 3 (A)

**Q 1:** Find $\mathcal{L}^{-1}\left\{\dfrac{2s-4}{(s^2+s)(s^2+1)}\right\}$.

**Sol:** Using partial fractions, we get

$$\frac{2s-4}{(s^2+s)(s^2+1)} = \frac{2s-4}{s(s+1)(s^2+1)} = \frac{-4}{s} + \frac{3}{s+1} + \frac{s+3}{s^2+1} = \frac{-4}{s} + \frac{3}{s+1} + \frac{s}{s^2+1} + \frac{3}{s^2+1}$$

$$\mathcal{L}^{-1}\left\{\frac{-4}{s} + \frac{3}{s+1} + \frac{s}{s^2+1} + \frac{3}{s^2+1}\right\} = -4 + 3e^{-t} + \cos(t) + 3\sin(t)$$

**Q 2:** Use laplace transform to solve $y'' - 6y' + 13y = 0,\ y(0) = 0,\ \ y'(0) = -3.$

**Sol:** Using partial fractions, we get

$$s^2Y(s) - sy(0) - y'(0) - 6sY(s) + 6y(0) + 13Y(s) = 0$$

$$(s^2 - 6s + 13)Y(s) = -3 \Rightarrow Y(s) = \frac{-3}{s^2 - 6s + 13} = \frac{-3}{(s-3)^2 + 2^2}$$

$$y(t) = -\tfrac{3}{2}e^{3t}\sin(2t).$$

**Q 3:** Find $\mathcal{L}^{-1}\left\{\dfrac{se^{-3s}}{(s^2+4)}\right\}$ and Find $\mathcal{L}\{\cos(t)u(t-\pi)\}$.

**Sol:** $\mathcal{L}^{-1}\left\{\dfrac{s}{(s^2+4)}e^{-3s}\right\} = \cos 2(t-3)u(t-3)$

$$\mathcal{L}\{\cos(t)u(t-\pi)\} = e^{-\pi s}\mathcal{L}\{\cos(t+\pi)\} = -e^{-\pi s}\mathcal{L}\{\cos(t)\} = -e^{-\pi s}\frac{s}{s^2+1}.$$

**Q 4:** Solve the Volterra Integral equation $f(t) = \cos(t) - \int_0^t e^{\tau} f(t-\tau)\, d\tau$.

**Sol:** $f(t) = \cos(t) - \int_0^t e^{\tau} f(t-\tau)\, d\tau = \cos(t) - e^t * f(t)$

Taking Laplace transform we get

$$F(s) = \frac{s}{s^2+1} - \frac{1}{s-1}F(s)$$

$$\Rightarrow \left(1 + \frac{1}{s-1}\right)F(s) = \frac{s}{s^2+1}$$

$$\Rightarrow F(s) = \frac{s-1}{s^2+1} = \frac{s}{s^2+1} + \frac{-1}{s^2+1}$$

$$\Rightarrow f(t) = \cos(t) - \sin(t).$$