Solution Math 301-103 Sec: 03 Quiz 2 (B)

Q.1: Evaluate $\int_C \overrightarrow{F} \cdot d\vec{r}$, where $\overrightarrow{F}(x,y,z) = yz\hat{i} + xz\hat{j} + xy\hat{k}$ and $\vec{r} = 3t\hat{i} - t^2\hat{j} + 2t\hat{k}$, $1 \leq t \leq 3$.

Sol: $x = 3t, y = -t^2, z = 2t$, $\overrightarrow{F}(t) = -2t^3\hat{i} + 6t^2\hat{j} - 3t^3\hat{k}$ and $d\vec{r} = 3\hat{i} - 2t\hat{j} + 2\hat{k}$.

$\int_C \overrightarrow{F} \cdot d\vec{r} = \int_1^3 (-6t^3 - 12t^3 - 6t^3)dt = \int_1^3 (-24t^3)dt = -6(3^4 - 1^4) = -6(80) = -480.$

Q.2: Evaluate $\int_{(3,-1,2)}^{(4,2,-3)} (yzdx + xzdy + xydz)$.

Sol: $yzdx + xzdy + xydz$ is an exact differential of $\phi(x,y,z) = xyz$

So $\int_{(3,-1,2)}^{(4,2,-3)} (yzdx + xzdy + xydz) = \phi(4,2,-3) - \phi(3,-1,2) = -24 + 6 = -18.$

Q.3: Evaluate $\oint_C (2x - 2y^2)dx + (x^2 - 3y)dy$, where C is the boundary of the region bounded by $x = y^2$ and $x = 4$.

Sol: $P(x,y) = 2x - 2y^2, Q(x,y) = x^2 - 3y$ and $\dfrac{\partial Q}{\partial x} = 2x, \dfrac{\partial P}{\partial y} = -4y$

The region R is the region bounded by parabola $x = y^2$ opens right and vertical line $x = 4$.

$\oint_C (2x - 2y^2)dx + (x^2 - 3y)dy = \iint_R (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dA = \int_{-2}^{2} \int_{y^2}^{4} (2x + 4y)dxdy = \frac{256}{5}.$