Math 301-102 Sec: 03 Solution Quiz 2 (B)

Q.1: Evaluate $\int_c (4x^2 + 6y^2)dx + 3xydy$, where C is given by $x = \sqrt{t}$, $y = t$, $9 \le t \le 16$.

Sol. $\int_9^{16} (4t + 6t^2)\frac{1}{2\sqrt{t}}dt + 3\sqrt{t}\, tdt = \int_9^{16} (2\sqrt{t} + 6t^{3/2})\, dt = \left[\frac{4}{3}t^{3/2} + \frac{12}{5}t^{5/2}\right]_9^{16}$

$$= (\tfrac{256}{3} + \tfrac{12288}{5}) - (36 + \tfrac{2916}{5}) = \tfrac{28856}{15}$$

Q.2: Determine whether the vector field $\mathbf{F}(x, y) = (4x^3y^3 + 5)\mathbf{i} + (3x^4y^2 + 2)\mathbf{j}$ is a gradient field. If so, find a potential function ϕ for $\mathbf{F}$.

Sol. $P(x, y) = 4x^3y^3 + 5$ and $Q(x, y) = 3x^4y^2 + 2$

$\dfrac{\partial P}{\partial y} = 12x^3y^2$ and $\dfrac{\partial Q}{\partial x} = 12x^3y^2$. Therefore the vector field is a gradient field. There exists a function ϕ such that $\mathbf{F} = grad\phi$. That is $P(x, y) = \dfrac{\partial \varphi}{\partial x}$ and $Q(x, y) = \dfrac{\partial \varphi}{\partial y}$.

$\dfrac{\partial \varphi}{\partial x} = 4x^3y^3 + 5 \Rightarrow \phi(x, y) = x^4y^3 + 5x + g(y)$

$\dfrac{\partial \varphi}{\partial y} = 3x^4y^2 + g'(y) = Q(x, y) = 3x^4y^2 + 2 \Rightarrow g'(y) = 2$ and $g(y) = 2y + C$

So $\phi(x, y) = x^4y^3 + 5x + 2y + C$ is the potential function.