Solution Math 301-101 Sec: 02 & 03 Quiz 2 (B)

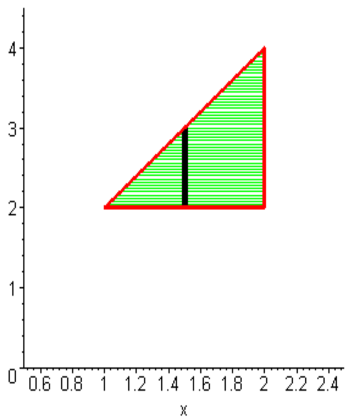
Q.1: Evaluate the integral l $\int_C 4xdx + 2ydy$, where C is given by $x = y^3 + 1$ from $(0, -1)$ to $(9, 2)$.

Sol: $\int_C 4xdx + 2ydy = \int_{-1}^{2} 4\left(y^3+1\right)3y^2dy + 2ydy = \int_{-1}^{2}\left(12y^5 + 12y^2 + 2y\right)dy$

$$= \left(\frac{12y^6}{6} + \frac{12y^3}{3} + y^2\right)\Big|_{-1}^{2} = 165.$$

Q.2: Use Green's theorem to evaluate the integral $\oint_C 2xydx + 3xy^2dy$, where C is the triangle $(1, 2)$, $(2, 2)$, $(2, 4)$.

Sol: $P(x, y) = 2xy,\ Q(x, y) = 3xy^2$ and $\frac{\partial Q}{\partial x} = 3y^2,\ \frac{\partial P}{\partial y} = 2x$



$$\oint_C 2xydx + 3xy^2dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA = \int_1^2\int_2^{2x}\left(3y^2 - 2x\right)dydx = \frac{56}{3}.$$

Q.3: Show that the integral $\int_{(0,0)}^{(2,8)}\left(3x^2y + y^3\right)dx + \left(x^3 + 3y^2x + 1\right)dy$ is independent of path. Find the function $\phi(x, y)$ such that $d\phi = \left(3x^2y + y^3\right)dx + \left(x^3 + 3y^2x\right)dy$. Use $\phi(x, y)$ to evaluate the integral.

Sol: $P(x, y) = 3x^2y + y^3,\ Q(x, y) = x^3 + 3y^2x + 1$ and $\frac{\partial Q}{\partial x} = 3x^2 + 3y^2 = \frac{\partial P}{\partial y}$. So the integral is independent of path. There exists a function $\phi(x, y)$ such that $d\phi(x, y) = P(x, y)dx + Q(x, y)dy$.

$$\frac{\partial \phi}{\partial x} = P = 3x^2y + y^3 \ \Rightarrow \phi(x, y) = x^3y + xy^3 + g(y)$$

$$\frac{\partial \phi}{\partial y} = x^3 + 3xy^2 + g'(y) = Q = x^3 + 3y^2x + 1 \ \Rightarrow g'(y) = 1 \ \Rightarrow g(y) = y.\ \text{So } \phi(x, y) = x^3y + xy^3 + y$$

$$\int_{(0,0)}^{(2,8)}\left(3x^2y + y^3\right)dx + \left(x^3 + 3y^2x + 1\right)dy = \int_{(0,0)}^{(2,8)} d\phi(x, y)$$

$$= \phi(x, y)\big|_{(0,0)}^{(2,8)} = \left(\left(2^3\right)(8) + 2\left(8^3\right) + 8\right) - 0 = 1096.$$