

## Solution Math 301-103 Sec: 03 Quiz 2 (A)

**Q.1:** Evaluate  $\int_C \vec{F} \cdot d\vec{r}$ , where  $\vec{F}(x, y, z) = yz\hat{i} + xz\hat{j} + xy\hat{k}$  and  $\vec{r} = t^2\hat{i} + 2t\hat{j} - t\hat{k}$ ,  $1 \leq t \leq 4$ .

**Sol:**  $x = t^2, y = 2t, z = -t$ ,  $\vec{F}(t) = -2t^2\hat{i} - t^3\hat{j} + 2t^3\hat{k}$  and  $d\vec{r} = 2t\hat{i} + 2\hat{j} - \hat{k}$ .

$$\int_C \vec{F} \cdot d\vec{r} = \int_1^4 (-4t^3 - 2t^3 - 2t^3) dt = \int_1^4 (-8t^3) dt = -2(4^4 - 1^4) = -2(255) = -510.$$

**Q.2:** Evaluate  $\int_{(1,1,1)}^{(2,3,4)} (yzdx + xzdy + xydz)$ .

**Sol:**  $yzdx + xzdy + xydz$  is an exact differential of  $\phi(x, y, z) = xyz$

$$\text{So } \int_{(1,1,1)}^{(2,3,4)} (yzdx + xzdy + xydz) = \phi(2, 3, 4) - \phi(1, 1, 1) = 24 - 1 = 23.$$

**Q.3:** Evaluate  $\oint_C (x + y^2)dx + (2x^2 - y)dy$ , where  $C$  is the boundary of the region bounded by  $y = x^2$  and  $y = 4$ .

**Sol:**  $P(x, y) = x + y^2, Q(x, y) = 2x^2 - y$  and  $\frac{\partial Q}{\partial x} = 4x, \frac{\partial P}{\partial y} = 2y$

The region  $R$  is the region bounded by parabola  $y = x^2$  opens up and horizontal line  $y = 4$ .

$$\oint_C (x + y^2)dx + (2x^2 - y)dy = \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_{-2}^2 \int_{x^2}^4 (4x - 2y) dy dx = \frac{-256}{5}.$$