Math 301-102 Sec: 03 Solution Quiz 2 (A)

Q.1: Evaluate $\int_c (6x^2 + 2y^2)dx + 4xydy$, where C is given by $x = \sqrt{t}$, $y = t$, $4 \le t \le 9$.

Sol. $\int_4^9 (6t + 2t^2)\frac{1}{2\sqrt{t}}dt + 4\sqrt{t}\ tdt = \int_4^9 (3\sqrt{t} + 5t^{3/2})\ dt = \left[2t^{3/2} + 2t^{5/2}\right]_4^9$

$$= (54 + 486) - (16 + 64) = 460$$

Q.2: Determine whether the vector field $\mathbf{F}(x, y) = (4x^3y^3 + 3)\mathbf{i} + (3x^4y^2 + 1)\mathbf{j}$ is a gradient field. If so, find a potential function ϕ for $\mathbf{F}$.

Sol. $P(x, y) = 4x^3y^3 + 3$ and $Q(x, y) = 3x^4y^2 + 1$

$\dfrac{\partial P}{\partial y} = 12x^3y^2$ and $\dfrac{\partial Q}{\partial x} = 12x^3y^2$. Therefore the vector field is a gradient field. There exists a function ϕ such that $\mathbf{F} = grad\phi$. That is $P(x, y) = \dfrac{\partial \varphi}{\partial x}$ and $Q(x, y) = \dfrac{\partial \varphi}{\partial y}$.

$\dfrac{\partial \varphi}{\partial x} = 4x^3y^3 + 3 \Rightarrow \phi(x, y) = x^4y^3 + 3x + g(y)$

$\dfrac{\partial \varphi}{\partial y} = 3x^4y^2 + g'(y) = Q(x, y) = 3x^4y^2 + 1 \Rightarrow g'(y) = 1$ and $g(y) = y + C$

So $\phi(x, y) = x^4y^3 + 3x + y + C$ is the potential function.