Solution Math 301-101 Sec: 02 & 03 Quiz 2 (A)

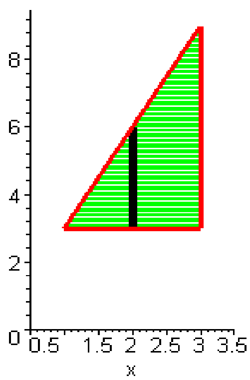
Q.1: Evaluate the integral $\int_C 2x^3y\,dx + (3x+y)\,dy$, where C is given by $x = y^2$ from $(1,-1)$ to $(1,1)$.

Sol: $\int_C 2x^3y\,dx + (3x+y)\,dy = \int_{-1}^{1} 2y^6y\ 2y\ dy + \left(3y^2+y\right)dy = \int_{-1}^{1}\left(4y^8+3y^2+y\right)dy$

$$\int_{-1}^{1}\left(4y^8+3y^2+y\right)dy = \left(\frac{4y^9}{9}+y^3+\frac{y^2}{2}\right)\Big|_{-1}^{1} = \frac{26}{9}.$$

Q.2: Use Green's theorem to evaluate the integral $\oint_C -3xy\,dx + 2xy^2\,dy$, where C is the triangle $(1,3)$, $(3,3)$, $(3,9)$.

Sol: $P(x,y) = -3xy,\ Q(x,y) = 2xy^2$ and $\frac{\partial Q}{\partial x} = 2y^2,\ \frac{\partial P}{\partial y} = -3x$



$$\oint_C -3xy\,dx + 2xy^2\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA = \int_1^3\int_3^{3x}\left(2y^2+3x\right)dy\,dx = 366$$

Q.3: Show that the integral $\int_{(1,2)}^{(3,6)}\left(2y^2x-3\right)dx + \left(2yx^2+4\right)dy$ is independent of path. Find the function $\phi(x,y)$ such that $d\phi = \left(2y^2x-3\right)dx + \left(2yx^2+4\right)dy$. Use $\phi(x,y)$ to evaluate the integral.

Sol: $P(x,y) = 2y^2x-3,\ Q(x,y) = 2yx^2+4$ and $\frac{\partial Q}{\partial x} = 4xy = \frac{\partial P}{\partial y}$. So the integral is independent of path. There exists a function $\phi(x,y)$ such that $d\phi(x,y) = P(x,y)dx + Q(x,y)\,dy$.

$$\frac{\partial \phi}{\partial x} = P = 2y^2x - 3 \Rightarrow \phi(x,y) = x^2y^2 - 3x + g(y)$$
$$\frac{\partial \phi}{\partial y} = 2x^2y + g'(y) = Q = 2x^2y+4 \Rightarrow g'(y) = 4 \Rightarrow g(y) = 4y.$$ So $\phi(x,y) = x^2y^2 - 3x + 4y$.

$$\int_{(1,2)}^{(3,6)}\left(2y^2x-3\right)dx + \left(2yx^2+4\right)dy = \int_{(1,2)}^{(3,6)} d\phi(x,y)$$

$$= \phi(x,y)|_{(1,2)}^{(3,6)} = \left(\left(3^2\right)\left(6^2\right) - 3(3) + 6(4)\right) - \left(\left(1^2\right)\left(2^2\right) - 3(1) + 4(2)\right) = 330.$$