Math 301-122 Quiz 1 (A)

Name:...Sec#:...............ID#:..............................Ser#:.........

Q.1: Evaluate $\int_0^4 \frac{1}{1+t^2}(3\hat{\mathbf{i}} - 3t\hat{\mathbf{j}} + t^2\hat{\mathbf{k}})dt$

$$= \int_0^4 3\frac{1}{1+t^2}dt\,\hat{i} - \frac{3}{2}\int_0^4 \frac{2t}{1+t^2}dt\,\hat{j} + \int_0^4 (1 - \frac{1}{1+t^2})dt\,\hat{k}$$

$$= 3\tan^{-1}t\Big|_0^4 \hat{i} - \frac{3}{2}Ln(1+t^2)\Big|_0^4 \hat{j} + (t - \tan^{-1}t)\Big|_0^4 \hat{k}$$

$$= 3\tan^{-1}4\,\hat{i} - \frac{3}{2}Ln\,17\,\hat{j} + (4 - \tan^{-1}4)\hat{k}$$

Q.2: Find the directional derivative of $f(x,y) = \frac{3xy}{x-y}$ at $(-1,-2)$ in the direction of the vector $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$

$$\nabla f = \frac{3y(x-y) - 3xy}{(x-y)^2}\hat{i} + \frac{3x(x-y) - 3xy(-1)}{(x-y)^2}\hat{j}$$

$$= \frac{-3y^2}{(x-y)^2}\hat{i} + \frac{3x^2}{(x-y)^2}\hat{j}$$

$$\nabla f(-1,-2) = -12\hat{i} + 3\hat{j}, \quad \hat{u} = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$$

$$D_{\hat{u}}f(-1,-2) = -\frac{36}{5} - \frac{12}{5} = -\frac{48}{5}$$

Q.3: Find the curl and divergence of $\overrightarrow{F}(x,y,z) = yz\ln x\hat{\mathbf{i}} + (2x - 3yz)\hat{\mathbf{j}} + xy^2z^3\hat{\mathbf{k}}$.

$$\nabla \cdot \vec{F} = \frac{yz}{x} - 3z + 3xy^2z^2$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yzLnx & 2x-3yz & xy^2z^3 \end{vmatrix}$$

$$= (2xyz^3 + 3y)\hat{i} + (yLnx - y^2z^3)\hat{j}$$

$$+ (2 - zLnx)\hat{k}$$

Math 301-122 Quiz 1 (B)

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Q.1: Evaluate $\int_0^5 \frac{1}{1+t^2}(-2\hat{\imath} + 4t\hat{\jmath} - 5t^2\hat{\mathrm{k}})dt$

$$\int_0^5 \frac{-2}{1+t^2}\,dt\,\hat{i} + 2\int_0^5 \frac{2t}{1+t^2}\,dt\,\hat{j} + (-5)\int_0^5 \left(1 - \frac{1}{1+t^2}\right)dt\,\hat{k}$$

$$= -2\tan^{-1}t\Big|_0^5\,\hat{i} + 2\ln(1+t^2)\Big|_0^5\,\hat{j} - 5(t - \cancel{\ } \tan^{-1}t)\Big|_0^5\,\hat{k}$$

$$= -2\tan^{-1}5 + 2\ln 26\,\hat{j} - 5(5 - \tan^{-1}5)\hat{k}$$

Q.2: Find the directional derivative of $f(x, y) = \frac{xy}{x+y}$ at $(2, -1)$ in the direction of the vector $6\hat{\imath} - 8\hat{\jmath}$

$$\nabla f = \frac{y(x+y) - xy}{(x+y)^2}\hat{i} + \frac{x(x+y) - xy}{(x+y)^2}\hat{j} = \frac{y^2\hat{i} + x^2\hat{j}}{(x+y)^2}$$

$$\nabla f(2,-1) = \frac{1}{1}\hat{i} + \frac{4}{1}\hat{j} = \hat{i} + 4\hat{j}$$

$$\hat{u} = \frac{6}{10}\hat{i} - \frac{8}{10}\hat{j} = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$$

$$D_{\hat{u}}f(2,-1) = \frac{3}{5} - \frac{16}{5} = -\frac{13}{5}$$

Q.3: Find the curl and divergence of $\vec{F}(x, y, z) = 2yz\ln x\hat{\imath} + (3x - 5yz)\hat{\jmath} + 3xy^2z^3\hat{\mathrm{k}}$.

$$\nabla\cdot\vec{F} = \frac{2yz}{x} + (-5z) + 9xy^2z^2 = \frac{2yz}{x} - 5z + 9xy^2z^2$$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2yz\ln x & 3x-5yz & 3xy^2z^3 \end{vmatrix}$$

$$= (6xyz^3 + 5y)\hat{i} + (2y\ln x - 3y^2z^3)\hat{j}$$

$$+ (3 - 2z\ln x)\hat{k}$$