Math 301-102 Sec: 03 Solution Quiz 1

Q.1: Find equation of tangent line to the curve $\overrightarrow{r}(t) = 3\cos t\ \hat{\mathbf{i}} + 3\sin t\ \hat{\mathbf{j}} + 2t\ \hat{\mathbf{k}}$ at $t = \frac{\pi}{4}$.

Sol. $\overrightarrow{r}'(t) = -3\sin t\ \hat{\mathbf{i}} + 3\cos t\ \hat{\mathbf{j}} + 2\ \hat{\mathbf{k}}$

At $t = \frac{\pi}{4}$, $\overrightarrow{r}\left(\frac{\pi}{4}\right) = 3\frac{\sqrt{2}}{2}\ \hat{\mathbf{i}} + 3\frac{\sqrt{2}}{2}\ \hat{\mathbf{j}} + \frac{\pi}{2}\ \hat{\mathbf{k}}$ and $\overrightarrow{r}'\left(\frac{\pi}{4}\right) = -3\frac{\sqrt{2}}{2}\ \hat{\mathbf{i}} + 3\frac{\sqrt{2}}{2}\ \hat{\mathbf{j}} + 2\ \hat{\mathbf{k}}$

Equation of the tangent line is: $\dfrac{x - 3\frac{\sqrt{2}}{2}}{-3\frac{\sqrt{2}}{2}} = \dfrac{y - 3\frac{\sqrt{2}}{2}}{3\frac{\sqrt{2}}{2}} = \dfrac{z - \frac{\pi}{2}}{2}$

Q.2: Find the directional derivative of $f(x, y) = x^2\tan(y)$ at $\left(\frac{1}{2}, \frac{\pi}{3}\right)$ in the direction of a vector with angle $\theta = \pi$.

Sol. The direction vector is $\hat{n} = \cos(\pi)\hat{\mathbf{i}} + \sin(\pi)\hat{\mathbf{j}} = -\hat{\mathbf{i}}$

$\mathit{gradf}\,(x, y) = \nabla f(x, y) = 2x\tan(y)\hat{\mathbf{i}} + x^2\sec^2(y)\hat{\mathbf{j}}$

At $\left(\frac{1}{2}, \frac{\pi}{3}\right)$, $\nabla f\left(\frac{1}{2}, \frac{\pi}{3}\right) = 2\frac{1}{2}\tan(\frac{\pi}{3})\hat{\mathbf{i}} + \frac{1}{4}\sec^2(\frac{\pi}{3})\hat{\mathbf{j}} = \sqrt{3}\hat{\mathbf{i}} + \hat{\mathbf{j}}$

The directional derivative is $D_{\hat{n}}f\left(\frac{1}{2}, \frac{\pi}{3}\right) = -\sqrt{3}$

Q.3: Find the curl and divergence of $\overrightarrow{F}(x, y, z) = x^2\sin(yz)\hat{\mathbf{i}} + z\cos(xz^3)\hat{\mathbf{j}} + ye^{5xy}\hat{\mathbf{k}}$

Sol. Divergence is: $\nabla \cdot \overrightarrow{F} = \dfrac{\partial P}{\partial x} + \dfrac{\partial Q}{\partial y} + \dfrac{\partial R}{\partial z} = 2x\sin(yz) + 0 + 0 = 2x\sin(yz)$.

Curl is: $\nabla \times \overrightarrow{F} = \begin{bmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ x^2\sin(yz) & z\cos(xz^3) & ye^{5xy} \end{bmatrix}$

$= [e^{5xy} + 5xye^{5xy} - \cos(xz^3) + 3xz^3\sin(xz^3)]\ \hat{\mathbf{i}}$

$+ [x^2y\cos(yz) - 5y^2e^{5xy}]\ \hat{\mathbf{j}}$

$+ [-z^4\sin(xz) - x^2z\cos(yz)]\ \hat{\mathbf{k}}$