Sloution Math 202 Quiz 4

Q.1: Solve the initial value problem using variation of parameters method,

$$y'' + 2y' - 8y = 2e^{-2x} - e^{-x}, \quad y(0) = 1, \quad y'(0) = 0.$$

Sol: Auxiliary equation is $m^2 + 2m - 8 = 0 \Rightarrow m = -4, 2$, and $y_c = Ae^{2x} + Be^{-4x}$.

Let $y_1 = e^{2x}$ and $y_2 = e^{-4x}$, then $W = \begin{vmatrix} e^{2x} & e^{-4x} \\ 2e^{2x} & -4e^{-4x} \end{vmatrix} = -6e^{-2x}$,

and $W_1 = \begin{vmatrix} 0 & e^{-4x} \\ 2e^{-2x} - e^{-x} & -4e^{-4x} \end{vmatrix} = e^{-5x} - 2e^{-6x}$, $W_2 = \begin{vmatrix} e^{2x} & 0 \\ 2e^{2x} & 2e^{-2x} - e^{-x} \end{vmatrix} = 2 - e^{x}$.

$$u_1(x) = \int \frac{e^{-5x} - 2e^{-6x}}{-6e^{-2x}} dx = \frac{1}{18}e^{-3x} - \frac{e^{-4x}}{12}, \; u_2(x) = \int \frac{2 - e^{x}}{-6e^{-2x}} dx = \frac{1}{18}e^{3x} - \frac{1}{6}e^{2x}$$

$$y_p = u_1(x) y_1(x) + u_2(x) y_2(x) = \frac{1}{18}e^{-x} - \frac{e^{-2x}}{12} + \frac{1}{18}e^{-x} - \frac{1}{6}e^{-2x} = \frac{1}{9}e^{-x} - \frac{1}{4}e^{-2x}$$

$$y = Ae^{2x} + Be^{-4x} + \frac{1}{9}e^{-x} - \frac{1}{4}e^{-2x} \text{ and } y' = 2Ae^{2x} - 4Be^{-4x} - \frac{1}{9}e^{-x} + \frac{1}{2}e^{-2x}$$

$$y(0) = 1 \Rightarrow A + B = 1 + \frac{1}{4} - \frac{1}{9} = \frac{41}{36} \text{ and } y'(0) = 0 \Rightarrow 2A - 4B = \frac{1}{9} - \frac{1}{2} = -\frac{7}{18}$$

$$6A = \frac{41}{9} - \frac{7}{18} = \frac{25}{6} \Rightarrow A = \frac{25}{36} \text{ and } B = \frac{41}{36} - \frac{25}{36} = \frac{4}{9}$$

$$y = y = \frac{25}{36}e^{2x} + \frac{4}{9}e^{-4x} + \frac{1}{9}e^{-x} - \frac{1}{4}e^{-2x}$$

Q.2: Solve $y'' + 3y' + 2y = 4x + 2$ using Undetermined coefficients (Annihilator Approach).

Sol: Auxiliary equation is $m^2 + 3m + 2 = 0 \Rightarrow m = -2, -1$, and $y_c = C_1e^{-x} + C_2e^{-2x}$.

In operator form, $(D^2 + 3D + 2) y = 4x + 2 \Rightarrow D^2 (D^2 + 3D + 2) y = D^2 (4x + 2) = 0$

Auxiliary equation is $m^2 (m^2 + 3m + 2) = 0 \Rightarrow m = -1, -2, 0, 0$

$y = C_1e^{-x} + C_2e^{-2x} + C_3 + C_4x$. Let $y_p = A + Bx$, then $y_p' = B$ and $y_p'' = 0$

$y_p'' + 3y_p' + 2y_p = 4x + 2 \Rightarrow 3B + 2A + 2Bx = 4x + 2 \Rightarrow B = 2$ and $A = -2$.

$y = C_1e^{-x} + C_2e^{-2x} - 2 + 2x$

Q.3: Find a differential operator that annihilates $2x^3 + 3x + 10x^2e^{-3x} + xe^{2x}\sin(5x)$.

Sol: $D^4 (2x^3 + 3x) = 0$, $(D + 3)^3 (10x^2e^{-3x}) = 0$, and $(D^2 - 4D + 29)^2 (xe^{2x}\sin(5x)) = 0$

So $D^4 (D + 3)^3 (D^2 - 4D + 29)^2 (2x^3 + 3x + 10x^2e^{-3x} + xe^{2x}\sin(5x)) = 0$.