Solution Math 202-091 Quiz 3 (A)

Q.1: If $y_1(x) = x+1$ is a solution of $(1-2x-x^2)\,y'' + 2(1+x)\,y' - 2y = 0$, then find a second solution $y_2(x)$.

Sol: Given equation can be written as $y'' - \dfrac{2+2x}{x^2+2x-1}y' + \dfrac{2}{x^2+2x-1}y = 0.$

$$y_2(x) = y_1(x)\int \frac{e^{-\int P(x)dx}}{(y_1(x))^2}dx = (x+1)\int \frac{e^{\int \frac{2x+2}{x^2+2x-1}dx}}{(x+1)^2}dx$$

$$= (x+1)\int \frac{e^{\ln\left(x^2+2x-1\right)}}{(x+1)^2}dx = (x+1)\int \frac{x^2+2x-1}{x^2+2x+1}dx$$

$$= (x+1)\int \left(1 - \frac{2}{x^2+2x+1}\right)dx = (x+1)\int \left(1 - \frac{2}{(x+1)^2}\right)dx$$

$$= (x+1)\left(x + \frac{2}{x+1}\right) = x^2 + x + 2.$$

Q.2: Solve the initial value problem $y''' + 12y'' + 36y' = 0, \quad y(0) = 0,\ y'(0) = 1,\ y''(0) = -7.$

Sol: Auxiliary equation is $m^3 + 12m^2 + 36m = 0 \ \Rightarrow m(m+6)^2 = 0 \ \Rightarrow m = 0,\ -6, -6.$

The solution is $y = Ae^{0x} + Be^{-6x} + Cxe^{-6x} = A + Be^{-6x} + Cxe^{-6x}$,

Then $y' = -6Be^{-6x} + Ce^{-6x} - 6Cxe^{-6x}$

and $y'' = 36Be^{-6x} - 6Ce^{-6x} - 6Ce^{-6x} + 36Cxe^{-6x} = 36Be^{-6x} - 12Ce^{-6x} + 36Cxe^{-6x}$

$y(0) = 0 \ \Rightarrow A + B = 0$

$y'(0) = 1 \ \Rightarrow -6B + C = 1$

$y''(0) = -7 \ \Rightarrow 36B - 12C = -7$

$C = \frac{1}{6},\ B = \frac{1}{6}C - \frac{1}{6} = \frac{1}{36} - \frac{6}{36} = -\frac{5}{36},\ A = \frac{5}{36}$

$y = \frac{5}{36} - \frac{5}{36}e^{-6x} + \frac{1}{6}xe^{-6x}.$