

SOLUTION Math 202-112

Quiz 2

(B)

Q.1: Solve the initial value problem $y^2 dx - 3(xy + y^5)dy = 0$, $y(1) = 1$. (Hint: check if it is linear in x or in y)

Sol: The equation is linear in x , $\frac{dx}{dy} - 3\frac{x}{y} = 3y^3$

$$\text{IF} = e^{\int \frac{-3}{y} dy} = e^{-3 \ln y} = y^{-3} = \frac{1}{y^3}$$

$$\frac{1}{y^3} \frac{dx}{dy} - 3 \frac{x}{y^4} = 3 \Rightarrow \frac{d}{dy} \left[x \frac{1}{y^3} \right] = 3 \Rightarrow x \frac{1}{y^3} = 3y + c \Rightarrow x = 3y^4 + cy^3$$

$$y(1) = 1 \Rightarrow 1 = 3 + c \Rightarrow c = -2$$

The solution is $x = 3y^4 - 2y^3$

Q.2: Solve the differential equation $\frac{dy}{dx} + \frac{\csc^2 y}{\cos^2 x} = 0$.

Sol: $\sin^2 y dy = -\sec^2 x dx$

$$\left(\frac{1 - \cos 2y}{2} \right) dy = -\sec^2 x dx$$

$$\frac{y}{2} - \frac{\sin 2y}{4} = -\tan x + c \Rightarrow 2y - \sin 2y = -4 \tan x + c$$

Q.3: Solve the differential equation $(4x + 3y^2)dx + (2xy)dy = 0$

Sol: $M(x, y) = 4x + 3y^2$ and $N(x, y) = 2xy$

$M_y = 6y$ and $N_x = 2y$, the equation is not exact.

$$\frac{M_y - N_x}{N} = \frac{6y - 2y}{2xy} = \frac{4y}{2xy} = \frac{2}{x} \text{ is function of } x \text{ alone}$$

$$\mu(x) = e^{\int \frac{2}{x} dx} = e^{2 \ln x} = x^2$$

Multiplying given equation by $\mu(x) = x^2$, we get $(4x^3 + 3x^2y^2)dx + (2x^3y)dy = 0$

$$\bar{M}(x, y) = 4x^3 + 3x^2y^2 \text{ and } \bar{N}(x, y) = 2x^3y$$

$\bar{M}_y = 6x^2y$ and $\bar{N}_x = 6x^2y$. Now the equation is exact.

There exists a function $f(x, y)$ such that

$$\frac{\partial f}{\partial y} = \bar{N}(x, y) = 2x^3y \Rightarrow f(x, y) = x^3y^2 + g(x)$$

$$\text{and } \frac{\partial f}{\partial x} = 3x^2y^2 + g'(x) = \bar{M}(x, y) = 4x^3 + 3x^2y^2 \Rightarrow g'(x) = 4x^3 \Rightarrow g(x) = x^4 + c$$

$f(x, y) = x^3y^2 + x^4 = c$ is the solution.