Math 202-102 Sec: 21 Solution Quiz 2 (B)

Q.1: Solve the initial value problem $(x+2)\dfrac{dy}{dx} + y = 2\ln(x)$, $y(1) = 1$.

Sol. We can write the given equation as $\dfrac{d}{dx}\left[(x+2)y\right] = 2\ln(x)$

Which $\Rightarrow (x+2)y = 2\int \ln(x)dx + c$, $\Rightarrow (x+1)y = 2x\ln(x) - 2x + c$ or $y = \dfrac{2x\ln(x) - 2x + c}{x+2}$

$y(1) = 1 \Rightarrow 1 = \dfrac{0 - 21 + c}{3} \Rightarrow c = 5$ and the solution is $y = \dfrac{2x\ln(x) - 2x + 5}{x+2}$.

Q.2: Solve the differential equation $6xydx + (4y + 9x^2)dy = 0$ by making it exact.

Sol. $M(x,y) = 6xy$ and $N(x,y) = 4y + 9x^2$. $\dfrac{\partial M}{\partial y} = 6x$ and $\dfrac{\partial N}{\partial x} = 18x$. So the equation is not exact.

$\dfrac{N_x - M_y}{M} = \dfrac{18x - 6x}{6xy} = \dfrac{2}{y}$ is a function of y alone.

$\mu(x) = e^{\int \frac{2}{y}dy} = e^{2\ln y} = y^2$ is the integrating factor.

Multiplying by integration factor, we get $6xy^3dx + (4y^3 + 9x^2y^2)dy = 0$

$\overline{M}(x,y) = 6xy^3$ and $\overline{N}(x,y) = 4y^3 + 9x^2y^2$. $\dfrac{\partial \overline{M}}{\partial y} = 184xy^2$ and $\dfrac{\partial \overline{N}}{\partial x} = 18xy^2$. So the equation is exact.

There exists a function $f(x,y)$ such that $\dfrac{\partial f}{\partial x} = \overline{M}$ and $\dfrac{\partial f}{\partial y} = \overline{N}$

$\dfrac{\partial f}{\partial x} = \overline{M} = 6xy^3, \quad \Rightarrow f(x,y) = 3x^2y^3 + g(y)$

$\dfrac{\partial f}{\partial y} = 9x^2y^2 + g'(y) = \overline{N} = 4y^3 + 9x^2y^2 \quad \Rightarrow g'(y) = 4y^3 \quad \Rightarrow g(y) = y^4$

So $f(x,y) = 3x^2y^3 + y^4 + c$

Q.3: (a.) Use appropriate substitution to transform $(y^2 + xy)dx + x^2dy = 0$ into a separable equation. Do not solve the new equation.

Sol. Given equation is a homogeneous equation of degree 2. Let $y = ux$, then $dy = udx + xdu$

$(y^2 + xy)dx + x^2dy = 0 \quad \Rightarrow (u^2x^2 + ux^2)dx + (x^2)(udx + xdu) = 0$

$u^2x^2dx + ux^2dx + x^2udx + x^3du = 0 \quad \Rightarrow x^2(u^2 + 2u)dx + x^3du = 0$

$\Rightarrow x^2(u^2 + 2u)dx = -x^3du \quad \Rightarrow \dfrac{1}{x}dx = \dfrac{-1}{u^2 + 2u}du$

Which is a separable equation.

(b.) Use appropriate substitution to transform $\dfrac{dy}{dx} - y = e^x y^2$ into a linear equation. Do not solve the new equation.

Sol. $\dfrac{dy}{dx} - y = e^x y^2$ is a Bernoulli equation with $n = 2$.

Let $u = y^{1-2} = y^{-1}$ or $y = u^{-1}$ and $\dfrac{dy}{dx} = -u^{-2}\dfrac{du}{dx}$

So $\dfrac{dy}{dx} - y = e^x y^2 \quad \Rightarrow -u^{-2}\dfrac{du}{dx} - u^{-1} = e^x u^{-2}$

Multiplying by $-u^2$ we get $\dfrac{du}{dx} + u = -e^x$ which is a linear equation.