SOLUTION Math 202-112 Quiz 2 (A)

Q.1: Solve the initial value problem $y^2dx - 2(xy + y^4)dy = 0$, $y(1) = 1$. (Hint: check if it is linear in x or in y)

Sol: The equation is linear in x, $\dfrac{dx}{dy} - 2\dfrac{x}{y} = 2y^2$

IF$= e^{\int \frac{-2}{y}dy} = e^{-2\ln y} = y^{-2} = \frac{1}{y^2}$

$\dfrac{1}{y^2}\dfrac{dx}{dy} - 2\dfrac{x}{y^3} = 2 \;\Rightarrow\; \dfrac{d}{dy}\left[x\dfrac{1}{y^2}\right] = 2 \;\Rightarrow\; x\dfrac{1}{y^2} = 2y + c \;\Rightarrow\; x = 2y^3 + cy^2$

$y(1) = 1 \;\Rightarrow\; 1 = 2 + c \;\Rightarrow\; c = -1$

The solution is $x = 2y^3 - y^2$

Q.2: Solve the differential equation $\dfrac{dy}{dx} + \dfrac{\sin^2 y}{\sec^2 x} = 0$.

Sol: $\dfrac{1}{\sin^2 y}dy = -\cos^2 x dx$

$\csc^2 y dy = -\left(\dfrac{1+\cos 2x}{2}\right)dx$

$-\cot y = -\left(\dfrac{x}{2} + \dfrac{\sin 2x}{4} + c\right) \;\Rightarrow\; 4\cot y = (2x + \sin 2x + c)$

Q.3: Solve the differential equation $(3x^2 + 2y^2)dx + (2xy)dy = 0$

Sol: $M(x,y) = 3x^2 + 2y^2$ and $N(x,y) = 2xy$

$M_y = 4y$ and $N_x = 2y$, the equation is not exact.

$\dfrac{M_y - N_x}{N} = \dfrac{4y - 2y}{2xy} = \dfrac{2y}{2xy} = \dfrac{1}{x}$ is function of x alone

$\mu(x) = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

Multiplying given equation by $\mu(x) = x$, we get $(3x^3 + 2xy^2)dx + (2x^2y)dy = 0$

$\bar{M}(x,y) = 3x^3 + 2xy^2$ and $\bar{N}(x,y) = 2x^2y$

$\bar{M}_y = 4xy$ and $\bar{N}_x = 2xy$. Now the equation is exact.

There exists a function $f(x,y)$ such that

$\dfrac{\partial f}{\partial y} = \bar{N}(x,y) = 2x^2y \;\Rightarrow\; f(x,y) = x^2y^2 + g(x)$

and $\dfrac{\partial f}{\partial x} = 2xy^2 + g'(x) = \bar{M}(x,y) = 3x^3 + 2xy^2 \;\Rightarrow\; g'(x) = 3x^3 \;\Rightarrow\; g(x) = \frac{3}{4}x^4 + c$

$f(x,y) = x^2y^2 + \frac{3}{4}x^4 = c$ is the solution.