Solution Math 202 Quiz 1 (B)

Q.1: Verify that $y = -x\cos(x) + \sin(x)\ln(\sin(x))$ is a solution of $y'' + y = \csc(x)$..

Sol:
$$y' = x\sin(x) - \cos(x) + \cos(x)\ln(\sin(x)) + \sin(x)\frac{\cos(x)}{\sin(x)} = x\sin x + \cos x\ln(\sin x)$$
$$y'' = x\cos(x) + \sin(x) - \sin(x)\ln(\sin(x)) + \cos(x)\frac{\cos(x)}{\sin(x)}$$
$$= [x\cos(x) - \sin(x)\ln(\sin(x))] + \sin(x) + \frac{\cos^2(x)}{\sin(x)} = -y + \frac{\cos^2(x) + \sin^2(x)}{\sin(x)}$$
$$y'' + y = \frac{1}{\sin(x)} = \csc(x).$$

Q.2: Determine a region of the $xy - plane$ for which the differential equation $\left(\sqrt{y^2-1}\right)y' = (x^2-2)(y^2-1)$ would have a unique solution containing the point (x_o, y_o).

Sol:
$$y' = \frac{(x^2-2)(y^2-1)}{\sqrt{y^2-1}} = (x^2-2)\sqrt{y^2-1}.$$
$f(x,y) = (x^2-2)\sqrt{y^2-1}$ and $\dfrac{\partial f}{\partial y} = \dfrac{y(x^2-2)}{\sqrt{y^2-1}}$.

$f(x,y)$ and $\dfrac{\partial f}{\partial y}$ are continuous for all $y > 1$ and $y < -1$.

Thus a region where the DE would have a unique solution is either $\{(x,y)\,|y > 1\}$ or $\{(x,y)\,|y < -1\}$

Q.3: Solve the differential equation $\sec^2(y)\,dx + \csc^2(x)\,dy = 0$.

Sol:
$$\csc^2(x)\,dy = -\sec^2(y)\,dx$$
$$\cos^2(y)\,dy = -\sin^2(x)\,dx$$
$$\left[\frac{1+\cos(2y)}{2}\right]dy = -\left[\frac{1-\cos(2x)}{2}\right]dx$$
$$\frac{1}{2}y + \frac{\sin(2y)}{4} = -\frac{1}{2}x + \frac{\sin(2x)}{4} + c$$
$$2y + \sin(2y) = -2x + \sin(2x).$$
$$2x + 2y = \sin(2x) - \sin(2y).$$