Solution Math 202 Quiz 1 (A)

Q.1: Verify that $y = x\sin(x) + \cos(x)\ln(\cos(x))$ is a solution of $y'' + y = \sec(x)$.

Sol:
$$y' = x\cos(x) + \sin(x) - \sin(x)\ln(\cos(x)) - \cos(x)\frac{\sin(x)}{\cos(x)} = x\cos x - \sin x\ln(\cos x)$$
$$y'' = -x\sin(x) + \cos(x) - \cos(x)\ln(\cos(x)) + \sin(x)\frac{\sin(x)}{\cos(x)}$$
$$= -[x\sin(x) + \cos(x)\ln(\cos(x))] + \cos(x) + \frac{\sin^2(x)}{\cos(x)} = -y + \frac{\cos^2(x) + \sin^2(x)}{\cos(x)}$$
$$y'' + y = \frac{1}{\cos(x)} = \sec(x).$$

Q.2: Determine a region of the $xy - plane$ for which the differential equation $\left(\sqrt{y^2-4}\right)y' = (x^2-4)(y^2-4)$ would have a unique solution containing the point (x_o, y_o).

Sol:
$$y' = \frac{(x^2-4)(y^2-4)}{\sqrt{y^2-4}} = (x^2-4)\sqrt{y^2-4}.$$
$f(x,y) = (x^2-4)\sqrt{y^2-4}$ and $\dfrac{\partial f}{\partial y} = \dfrac{y(x^2-4)}{\sqrt{y^2-4}}$.

$f(x,y)$ and $\dfrac{\partial f}{\partial y}$ are continuous for all $y > 2$ and $y < -2$.

Thus a region where the DE would have a unique solution is either $\{(x,y)\,|y>2\}$ or $\{(x,y)\,|y<-2\}$

Q.3: Solve the differential equation $\csc^2(y)\,dx + \sec^2(x)\,dy = 0$.

Sol:
$$\sec^2(x)\,dy = -\csc^2(y)\,dx$$
$$\sin^2(y)\,dy = -\cos^2(x)\,dx$$
$$\left[\frac{1-\cos(2y)}{2}\right]dy = -\left[\frac{1+\cos(2x)}{2}\right]dx$$
$$\frac{1}{2}y - \frac{\sin(2y)}{4} = -\frac{1}{2}x - \frac{\sin(2x)}{4} + c$$
$$2y - \sin(2y) = -2x - \sin(2x).$$
$$2x + 2y = -\sin(2x) + \sin(2y).$$