Sol Math 201-111 Quiz 8

Q.1: Evaluate the triple integral $\int\int\int_E \sqrt{x^2+y^2}dV$, where E is the region that lies inside the cylinder $x^2+y^2=16$ and between the planes $z=-5$ and $z=4$.

Sol: $\int\int\int_E \sqrt{x^2+y^2}dV = \int_0^{2\pi}\int_0^4\int_{-5}^4 r^2 dz dr d\theta = \int_0^{2\pi}\int_0^4 9r^2 dr d\theta = (2\pi)(3)(4^3) = 384\pi$

Q.2: Evaluate the triple integral $\int\int\int_E x^2 dV$,

where E is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$ and below the cone $z^2=4x^2+4y^2$.

Sol: $\int\int\int_E x^2 dV = \int_0^{2\pi}\int_0^1\int_0^{2r} r^3\cos^2\theta dz dr d\theta = \int_0^{2\pi}\int_0^1 2r^4\cos^2\theta dr d\theta = \frac{2}{5}\int_0^{2\pi}\frac{1+\cos 2\theta}{2}d\theta$

$= \frac{1}{5}(\theta + \frac{\sin 2\theta}{2})|_0^{2\pi} = \frac{2\pi}{5}$