Sol Math 201-111 Quiz 7

Q.1: Evaluate the triple integral $\int\int\int_E zdV$, where E is the solid tetrahedron bounded by the coordinate planes and the plane $x+y+z=1$.

Sol: $\int\int\int_E zdV = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} zdzdydx = \int_0^1 \int_0^{1-x} \frac{(1-x-y)^2}{2} dydx$

$$= \int_0^1 -\frac{(1-x-y)^3}{6}|_0^{1-x} dx = \int_0^1 \frac{(1-x)^3}{6} dx = -\frac{(1-x)^4}{24}|_0^1 = \frac{1}{24}.$$

Q.2: Evaluate the triple integral $\int\int\int_E x^2 e^y dV$,

where E is bounded by $z=1-y^2$, and planes $z=0,\ x=1,\ x=-1$.

Sol: $\int\int\int_E x^2 e^y dV = \int_{-1}^1 \int_{-1}^1 \int_0^{1-y^2} x^2 e^y dzdxdy = \int_{-1}^1 \int_{-1}^1 x^2 e^y (1-y^2) dxdy = \frac{2}{3}\int_{-1}^1 e^y(1-y^2)dy$

$$= \frac{2}{3}\left[e^y(1-y^2)\right]|_{-1}^1 + \frac{2}{3}\int_{-1}^1 2ye^y dy = \frac{4}{3}\left[(ye^y - e^y)\right]|_{-1}^1$$

$$= \frac{4}{3}\left[(e^1 - e^1 + e^{-1} + e^{-1}\right] = \frac{8}{3e}$$