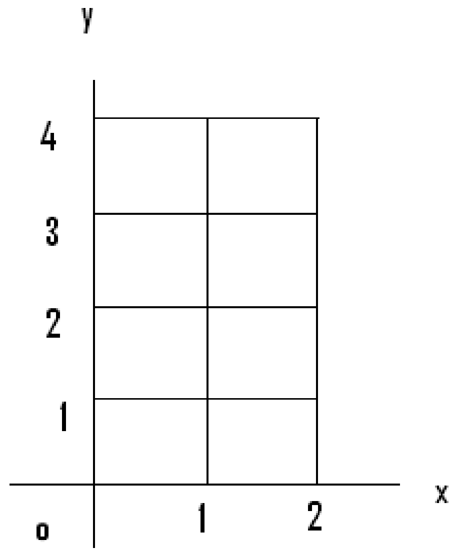
Q.1: Estimate volume of the solid that lies below the surface $z = 2x^2 + y^2$ and above the rectangle $R = [0,2] \times [0,4]$. Use a Riemann sum with $m = 2$ and $n = 4$ and choose upper right corner points.

Sol: Here $f(x,y) = x^2 + 2y^2$.



The lower left points are $(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)$ and $\Delta A = \Delta x \Delta y = 1$.

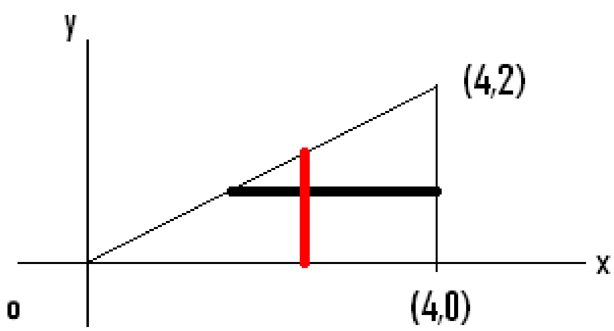
$V \approx \Delta A [f(1,1) + f(1,2) + f(1,3) + f(1,4) + f(2,1) + f(2,2) + f(2,3) + f(2,4)] = 3 + 6 + 11 + 18 + 9 + 12 + 17 + 24 = 100$.

Q.2: Find exact volume of the solid that lies below the surface $z = 2x^2 + y^2$ and above the rectangle $R = [0,2] \times [0,4]$.

Sol: $V = \int_0^4 \int_0^2 (2x^2 + y^2)\, dxdy = \int_0^4 \left(\frac{2x^3}{3} + xy^2 \right)\Big|_0^2 dy = \int_0^4 \left(\frac{16}{3} + 2y^2 \right) dy = \frac{16}{3}y + \frac{2y^3}{3}\Big|_0^4 = \frac{64}{3} + \frac{128}{3} = \frac{192}{3} = 64.$

Q.3: Evaluate the double integral $\int_0^2 \int_{2y}^4 6e^{x^2}\, dxdy$. (Hint: Change order of integration)

Sol: The region is $0 \le y \le 2,\ 2y \le x \le 4$.



$$\int_0^4 \int_0^{\frac{x}{2}} 6e^{x^2}\, dydx = \int_0^4 3xe^{x^2}\, dx = \frac{3}{2}e^{x^2}\Big|_0^4 = \frac{3}{2}\left(e^{16} - 1\right).$$

Q.4: Use double integral to find the area enclosed by one loop of the graph of $r = \cos(3\theta)$.

Sol: $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \int_0^{\cos(3\theta)} r\, drd\theta = \frac{1}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} r^2\Big|_0^{\cos(3\theta)} d\theta = \frac{1}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(3\theta)\, d\theta = \frac{1}{2} \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1 + \cos(6\theta)}{2} d\theta = \frac{1}{4}\left(\theta + \frac{\sin(6\theta)}{6} \right)\Big|_{-\frac{\pi}{6}}^{\frac{\pi}{6}} = \frac{\pi}{12}.$