Solution Math 201 Quiz 6 (B)

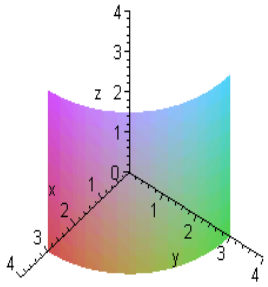
Q.1: Evaluate $\iiint_E y^2 dV$, where E is the region bounded by the parabolic cylinder $z = 4 - x^2$ and the palnes $z = 0$, $y = 2$, and $y = -2$.

Sol: $$\iiint_E y^2 dV = \int_{-2}^{2}\int_{-2}^{2}\int_{0}^{4-x^2} y^2 dz dx dy = \int_{-2}^{2}\int_{-2}^{2}\int_{0}^{4-x^2} y^2\left(4-x^2\right) dx dy = \int_{-2}^{2} y^2\left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2} dy$$
$$= \frac{32}{3}\int_{-2}^{2} y^2 dy = \frac{512}{9}.$$

Q.2: Evaluate $\iiint_E dV$, where E is the region bounded by the coordinate planes and $3x - 2y + z = 5$.

Sol: $$\iiint_E dV = \int_{0}^{\frac{5}{3}}\int_{0}^{\frac{3x-5}{2}}\int_{0}^{5-3x+2y} dz dy dx = \int_{0}^{\frac{5}{3}}\int_{0}^{\frac{3x-5}{2}} (5-3x+2y)\, dy dx = \int_{0}^{\frac{5}{3}} \left(5y - 3xy + y^2\right)\Big|_0^{\frac{3x-5}{2}} dx$$
$$\int_{0}^{\frac{5}{3}} \left(5\left(\frac{3x-5}{2}\right) - 3x\left(\frac{3x-5}{2}\right) + \left(\frac{3x-5}{2}\right)^2\right) dx = \int_{0}^{\frac{5}{3}} \left(\frac{15}{2}x - \frac{9}{4}x^2 - \frac{25}{4}\right) dx. = -\frac{125}{36}.$$

Q.3: Setup tripple integral for the given figure and evaluate its volume.



Sol: $$\int_{0}^{\frac{\pi}{2}}\int_{0}^{3}\int_{0}^{3} r dz dr d\theta = \frac{\pi}{2}.3.\left.\frac{r^2}{2}\right|_0^3 = \frac{27\pi}{4}.$$

Q.4: Evaluate $\iiint_E dV$, where E is the region bounded by the spheres $x^2+y^2+z^2 = 9$ and $x^2+y^2+z^2 = 16$.

Sol: $$\iiint_E dV = \int_{0}^{2\pi}\int_{0}^{\pi}\int_{3}^{4} \rho^2 \sin(\phi)\, d\rho d\phi d\theta = \int_{0}^{2\pi} d\theta \int_{0}^{\pi} \sin(\phi)\, d\rho d\phi \int_{3}^{4} \rho^2 d\rho = \frac{148}{3}\pi.$$