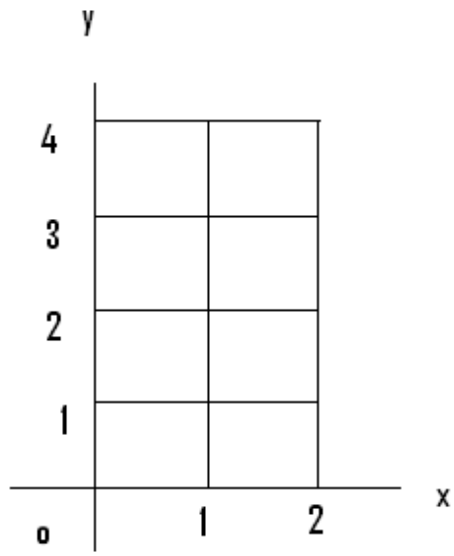
Math 201 Quiz 6 (A)

Name:.....Serial#:.....Sec #.....

Q.1: Estimate volume of the solid that lies below the surface $z = x^2 + 2y^2$ and above the rectangle $R = [0,2] \times [0,4]$. Use a Riemann sum with $m = 2$ and $n = 4$ and choose lower left corner points.

Sol: Here $f(x,y) = x^2 + 2y^2$.



The lower left points are $(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (1,3)$ and $\Delta A = \Delta x \Delta y = 1$.

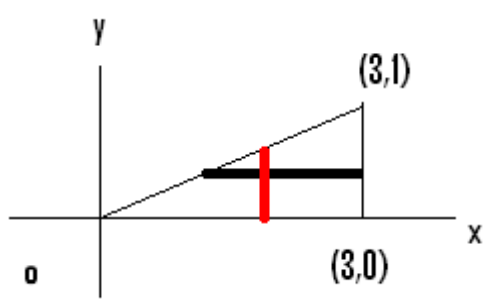
$V \approx \Delta A\left[f(0,0) + f(0,1) + f(0,2) + f(0,3) + f(1,0) + f(1,1) + f(1,2) + f(1,3)\right] = 0 + 2 + 8 + 18 + 1 + 3 + 9 + 19 = 60$.

Q.2: Find exact volume of the solid that lies below the surface $z = x^2 + 2y^2$ and above the rectangle $R = [0,2] \times [0,4]$.

Sol: $V = \int_0^4 \int_0^2 \left(x^2 + 2y^2\right) dxdy = \int_0^4 \left(\frac{x^3}{3} + 2xy^2\right)\Big|_0^2 dy = \int_0^4 \left(\frac{8}{3} + 4y^2\right) dy = \frac{8}{3}y + \frac{4y^3}{3}\Big|_0^4 = \frac{32}{3} + \frac{256}{3} = \frac{288}{3} = 96$.

Q.3: Evaluate the double integral $\int\limits_0^1 \int\limits_{3y}^3 4e^{x^2} dxdy$. (Hint: Change order of integration)

Sol: The region $0 \le y \le 1$ and $3y \le x \le 3$ is



$$\int\limits_0^3 \int\limits_0^{\frac{x}{3}} 4e^{x^2} dydx = \int\limits_0^3 \frac{4}{3} x e^{x^2} dx = \frac{2}{3} e^{x^2}\Big|_0^3 = \frac{2}{3}\left(e^9 - 1\right).$$

Q.4: Use double integral to find the area enclosed by one loop of the graph of $r = \cos(2\theta)$.

Sol: $\int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \int\limits_0^{\cos(2\theta)} r dr d\theta = \frac{1}{2} \int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} r^2\Big|_0^{\cos(2\theta)} d\theta = \frac{1}{2} \int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \cos^2(2\theta)\, d\theta = \frac{1}{2} \int\limits_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1 + \cos(4\theta)}{2} d\theta = \frac{1}{4}\left(\theta + \frac{\sin(4\theta)}{4}\right)\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \frac{\pi}{8}$.