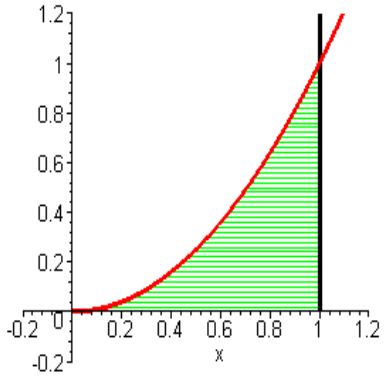
Solution Math 201 Quiz 6 (A)

Q.1: Evaluate the double integral $\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}dxdy$. (Hint: Sketch the region and reverse the order of integration)

Sol: The region is $\sqrt{y} \le x \le 1, 0 \le y \le 1$.



$$\int_0^1 \int_0^{x^2} \sqrt{x^3+1}dydx = \frac{1}{3}\int_0^1 3x^2\sqrt{x^3+1}dx = \frac{1}{3}\frac{2}{3}\left(x^3+1\right)^{\frac{3}{2}}\Big|_0^1 = \frac{2}{9}\left(2\sqrt{2}-1\right).$$

Q.2: Evaluate the double integral $\iint_D \frac{2y}{\sqrt{x^3+1}}dA,\ D = \{(x,y) \mid 2 \le x \le 3, 0 \le y \le 3x\}$.

Sol: $$\iint_D \frac{2y}{\sqrt{x^3+1}}dA = \int_2^3 \int_0^{3x} \frac{2y}{\sqrt{x^3+1}}dydx = \int_2^3 \frac{y^2}{\sqrt{x^3+1}}\Big|_{y=0}^{y=3x} dx$$

$$= \int_2^3 9x^2\left(x^3+1\right)^{-\frac{1}{2}}dx = 6\left(x^3+1\right)^{\frac{1}{2}}\Big|_2^3 = 12\sqrt{7}-18$$