Solution Math 201 Quiz 6 (A)

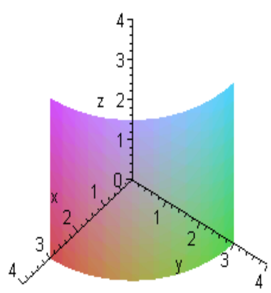
Q.1: Evaluate $\iiint\limits_E x^2 dV$, where E is the region bounded by the parabolic cylinder $z = 1 - y^2$ and the palnes $z = 0$, $x = 1$, and $x = -1$.

Sol: $$\iiint\limits_E x^2 dV = \int_{-1}^{1}\int_{-1}^{1}\int_{0}^{1-y^2} x^2 dzdydx = \int_{-1}^{1}\int_{-1}^{1} x^2\left(1-y^2\right)dydx = \int_{-1}^{1} x^2\left(y - \frac{y^3}{3}\right)\Big|_{-1}^{1} dx = \frac{4}{3}\int_{-1}^{1} x^2 dx = \frac{8}{9}.$$

Q.2: Evaluate $\iiint\limits_E dV$, where E is the region bounded by the coordinate planes and $2x + 2y + z = 4$.

Sol: $$\iiint\limits_E dV = \int_{0}^{2}\int_{0}^{2-x}\int_{0}^{4-2x-2y} dzdydx = \int_{0}^{2}\int_{0}^{2-x}(4-2x-2y)\,dydx = \int_{0}^{2}\left(4y - 2xy - y^2\right)\Big|_0^{2-x} dx$$
$$\int_0^2\left(4(2-x) - 2x(2-x) - (2-x)^2\right)dx = \int_0^2\left(8 - 4x - 4x + 2x^2 - \left(4 - 4x + x^2\right)\right)dx = \frac{8}{3}.$$

Q.3: Setup tripple integral for the given figure and evaluate its volume.



Sol: $$\int_{0}^{\frac{\pi}{2}}\int_{0}^{3}\int_{0}^{3} r dzdrd\theta = \frac{\pi}{2}.3.\left.\frac{r^2}{2}\right|_0^3 = \frac{27\pi}{4}.$$

Q.4: Evaluate $\iiint\limits_E dV$, where E is the region bounded by the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$.

Sol: $$\iiint\limits_E dV = \int_{0}^{2\pi}\int_{0}^{\pi}\int_{1}^{2} \rho^2\sin(\phi)\,d\rho d\phi d\theta = \int_0^{2\pi} d\theta\int_0^{\pi}\sin(\phi)\,d\rho d\phi\int_1^2 \rho^2 d\rho = \frac{28}{3}\pi.$$