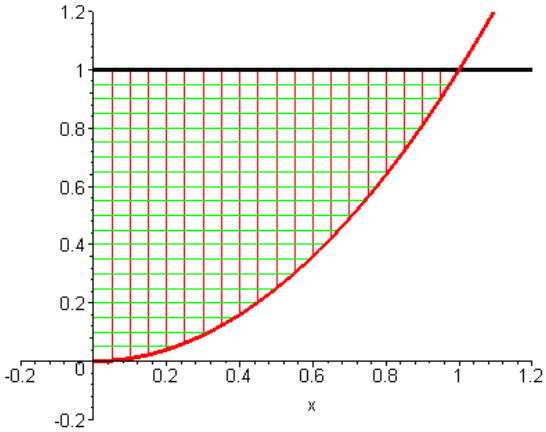
Solution Math 201 Quiz 5 (B)

Q.1: Evaluate the integral $\int_0^1 \int_{x^2}^1 x^3 \cos\left(y^3\right) dydx$

Sol: We need to change the order of integration



$$\int_0^1 \int_0^{\sqrt{y}} x^3 \cos\left(y^3\right) dxdy = \int_0^1 \frac{x^4}{4}\Big|_0^{\sqrt{y}} \cos\left(y^3\right) dy = \frac{1}{4}\int_0^1 \mathrm{y}^2 \cos\left(y^3\right) dy = \frac{\sin\left(y^3\right)}{12}\Big|_0^1 = \frac{\sin(1)}{12}$$

Q.2: Find volume of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 3y,\ x = 0, z = 0$ in the first octant.

Sol: $\int_0^3 \int_0^{\sqrt{9-x^2}} 3ydydx = \int_0^3 3\,\frac{y^2}{2}\Big|_0^{\sqrt{9-x^2}} dx = \frac{3}{2}\int_0^3 \left(9 - x^2\right) dx = \frac{3}{2}\left(9x - \frac{x^3}{3}\right)\Big|_0^3 = 27.$

Q.3: Evaluate the integral $\iint_R e^{-\left(x^2+y^2\right)} dA$, where R is the region that lies above the $x - axis$ and within the circle $x^2 + y^2 = 9$.

Sol: Using polar coordinates

$$\iint_R e^{-\left(x^2+y^2\right)} dA = \int_0^\pi \int_0^3 e^{-r^2} r dr d\theta = \int_0^\pi d\theta \int_0^3 \frac{-1}{2}\left(-2r\right) e^{-r^2} dr = \frac{-\pi}{2}\left(e^{-r^2}\right)\Big|_0^3 = \frac{\pi}{2}\left(1 - e^{-9}\right).$$