Math 201 Quiz 5 (A)

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Q.1: Find the point on the plane $x+2y+z=4$ that is closest to the point $(1,0,-2)$. Also find the shortest distance of $(1,0,-2)$ from the plane.

Sol: Let $P(x,y,z)$ be any point on the plane. Its distance from the point $(1,0,-2)$ is $d=\sqrt{(x-1)^2+y^2+(z+2)^2}$.

From equation of the plane, we get $z=4-x-2y$ and rewrite $d=\sqrt{(x-1)^2+y^2+(6-x-2y)^2}$.

Define $f(x,y)=d^2=(x-1)^2+y^2+(6-x-2y)^2$. We need to minimize the function $f(x,y)$.

$f_x=2(x-1)-2(6-x-2y)=4x+4y-14$ and $f_y=2y-4(6-x-2y)=4x+10y-24$.

$f_x=0$ and $f_y=0 \Rightarrow x=\dfrac{11}{6}$ and $y=\dfrac{10}{6}$.

Now $f_{xx}=4,\ f_{yy}=10,\ f_{xy}=4$, and discriminant $D=f_{xx}f_{yy}-(f_{xy})^2=40-16=24>0$. Also $f_{xx}=4>0$.

$\Rightarrow f(x,y)$ is minimum at $\left(\dfrac{11}{6},\dfrac{10}{6}\right)$, that is, d is minimum.

Also $z=4-x-2y=4-\dfrac{11}{6}-\dfrac{20}{6}=-\dfrac{7}{6}\Rightarrow\left(\dfrac{11}{6},\dfrac{10}{6},\dfrac{-7}{6}\right)$ is on the plane $x+2y+z=4$ and is closest to the point $(1,0,-2)$.

The shortest distance is $d=\sqrt{\left(\dfrac{11}{6}-1\right)^2+\left(\dfrac{10}{6}\right)^2+\left(\dfrac{-7}{6}+2\right)^2}=\dfrac{5}{6}\sqrt{6}$.

Q.2: Find the local maximum and local minimum values of the function $f(x,y)=9-2x+4y-x^2-2y^2$.

Sol: $f_x=-2x-2,\ f_y=-4y+4$ and $f_x=0,\ f_y=0 \Rightarrow x=-1,\ y=1$.

$f_{xx}=-2,\ f_{yy}=-4,\ f_{xy}=0$. Thus $D=f_{xx}f_{yy}-(f_{xy})^2=8>0$, and $f_{xx}=-2<0\Rightarrow f$ ha a local maximum at $(-1,1)$.

Q.3: Use Lagrange Multipliers to find the maximum and minimum values of the function $f(x,y)=x^2-y^2$ subject to the constraint $x^2+y^2=1$.

Sol: Define $g(x,y)=x^2+y^2$. Then $\nabla f=\lambda\nabla g \Rightarrow 2x=2x\lambda$ and $-2y=2y\lambda$.

$2x=2x\lambda \Rightarrow x(1-\lambda)=0 \Rightarrow x=0$ or $\lambda=1$. For $x=0,\ g(x,y)=1 \Rightarrow y=\pm1$. So the points are $(0,\pm1)$.

$-2y=2y\lambda \Rightarrow y(1+\lambda)=0 \Rightarrow y=0$ or $\lambda=-1$. For $y=0,\ g(x,y)=1 \Rightarrow x=\pm1$. So the points are $(\pm1,0)$.

Now $f(0,\pm1)=-1$ and $f(\pm1,0)=1 \Rightarrow$maximum value of f subject to the constraint $x^2+y^2=1$ is 1 and minimum value is -1.