Solution Math 201-103 Sec: 07 Quiz 4 (B)

Q.1: Find the domain and range of $f(x, y) = \arcsin(x^2 + y^2 - 8)$. Also sketch the domain.

Sol: For domain $-1 \leq x^2 + y^2 - 8 \leq 1$ which implies $7 \leq x^2 + y^2 \leq 9$. So the domain is two concentric circles of radii $\sqrt{7}$ and 3.

$D = \{(x, y)|7 \leq x^2 + y^2 \leq 9\}$ and the range is $-\frac{\pi}{2} \leq f(x, y) \leq \frac{\pi}{2}$.

Q.2: Find the limit $\lim_{(x,y)\to(0,0)} \frac{3xy^2}{3x^2 + 2y^4}$ if exist or show that limit does not exist.

Sol: For $x = 0$ and $y \to 0$, $\lim_{y\to 0} \frac{3(0)y^2}{3(0)^2 + 2y^4} = 0$

For $x = y^2$, $\lim_{y\to 0} \frac{3y^2y^2}{3y^4 + 2y^4} = \lim_{x\to 0} \frac{3y^4}{5y^4} = \frac{3}{5}$.

So the limit does not exist.

Q.3: For $f(x, y) = x^2ye^{xy^3}$, find $\frac{\partial^3 f}{\partial x\partial y\partial x}$.

Sol: $\frac{\partial f}{\partial x} = (2xy + x^2y^4)e^{xy^3}$, $\frac{\partial^2 f}{\partial y\partial x} = (2x + 10x^2y^3 + 3x^3y^6)e^{xy^3}$

$\frac{\partial^3 f}{\partial x\partial y\partial x} = (2 + 22xy^3 + 19x^2y^6 + 3x^3y^9)e^{xy^3}$