Solution Math 201-111 Quiz 4 (A)

Q.1: Find points on the sphere $x^2+y^2+z^2=4$ that are closest to and farthest from the point $(3,1,-1)$.

Sol: Distance of point $(3,1,-1)$ from the $x^2+y^2+z^2=4$ is given by

$D=\sqrt{(x-3)^2+(y-1)^2+(z+1)^2}$.

Let $f(x,y,z)=D^2=(x-3)^2+(y-1)^2+(z+1)^2$ and $g(x,y,z)=x^2+y^2+z^2=4$,

then $\nabla f=\lambda\nabla g \;\Rightarrow\; 2(x-3)=2\lambda x,\ 2(y-1)=2\lambda y,\ 2(z+1)=2\lambda z,$

$\Rightarrow x=\frac{3}{1-\lambda},\ y=\frac{1}{1-\lambda},\ z=\frac{-1}{1-\lambda}$

$x^2+y^2+z^2=4 \;\Rightarrow\; \frac{9}{(1-\lambda)^2}+\frac{1}{(1-\lambda)^2}+\frac{1}{(1-\lambda)^2}=4 \;\Rightarrow 1-\lambda=\pm\frac{\sqrt{11}}{2}$

For $1-\lambda=\frac{\sqrt{11}}{2},\ x=\frac{6}{\sqrt{11}},\ y=\frac{2}{\sqrt{11}},\ x=\frac{-2}{\sqrt{11}}$

For $1-\lambda=-\frac{\sqrt{11}}{2},\ x=\frac{-6}{\sqrt{11}},\ y=\frac{-2}{\sqrt{11}},\ x=\frac{2}{\sqrt{11}}$

The point on the sphere that is closest to $(3,1,-1)$ is $(\frac{6}{\sqrt{11}},\frac{2}{\sqrt{11}},\frac{-2}{\sqrt{11}})$

The point on the sphere that is farthest from $(3,1,-1)$ is $(-\frac{6}{\sqrt{11}},-\frac{2}{\sqrt{11}},\frac{2}{\sqrt{11}})$

Q.2: Use midpoint rule with $m=3$ and $n=3$ to approximate $\int\int_R(2x-3y)dA$, where $R=\{(x,y)|0\le x\le 6, 0\le y\le 3\}$

Sol: $\Delta A=\Delta x\Delta y=(2)(1)$,

The points are $(1,\frac{1}{2}),(1,\frac{3}{2}),(1,\frac{5}{2}),(3,\frac{1}{2}),(3,\frac{3}{2}),(3,\frac{5}{2}),(5,\frac{1}{2}),(5,\frac{3}{2}),(5,\frac{5}{2})$

$$\int\int_R(2x-3y)dA\approx[2-\tfrac{3}{2}+2-\tfrac{9}{2}+2-\tfrac{15}{2}+6-\tfrac{3}{2}+6-\tfrac{9}{2}+6-\tfrac{15}{2}$$

$$+10-\tfrac{3}{2}+10-\tfrac{9}{2}+10-\tfrac{15}{2}]=27$$

Q.3: Evaluate $\int\int_R\frac{xy^2}{1+x^2}dA$, where $R=\{(x,y)|0\le x\le 1,-3\le y\le 3\}$

Sol: $\int\int_R\frac{xy^2}{1+x^2}dA=\int_{-3}^{3}\int_0^1\frac{xy^2}{1+x^2}dxdy=\frac{1}{2}\int_{-3}^{3}y^2\ln 1+x^2|_0^1dy=\frac{1}{2}\ln 2\int_{-3}^{3}y^2dy=9\ln 2$