Solution Math 201-103 Sec: 07 Quiz 4 (A)

Q.1: Find the domain and range of $f(x,y) = \arcsin(x^2+y^2-3)$. Also sketch the domain.

Sol: For domain $-1 \le x^2+y^2-3 \le 1$ which implies $2 \le x^2+y^2 \le 4$. So the domain is two concentric circles of radii $\sqrt{2}$ and 2.

$D = \{(x,y)|2 \le x^2+y^2 \le 4\}$ and the range is $-\dfrac{\pi}{2} \le f(x,y) \le \dfrac{\pi}{2}$.

Q.2: Find the limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{2x^2y}{3x^4+2y^2}$ if exist or show that limit does not exist.

Sol: For $x = 0$ and $y \to 0$, $\lim\limits_{y\to 0} \dfrac{20^2y}{30^4+2y^2} = 0$

For $y = x^2$, $\lim\limits_{x\to 0} \dfrac{2x^2x^2}{3x^4+2x^4} = \lim\limits_{x\to 0} \dfrac{2x^4}{5x^4} = \dfrac{2}{5}$.

So the limit does not exist.

Q.3: For $f(x,y) = x^2ye^{xy^3}$, find $\dfrac{\partial^3 f}{\partial y \partial x \partial y}$.

Sol: $\dfrac{\partial f}{\partial y} = (x^2+3x^3y^3)e^{xy^3}$, $\dfrac{\partial^2 f}{\partial x \partial y} = (2x+10x^2y^3+3x^3y^6)e^{xy^3}$

$\dfrac{\partial^3 f}{\partial y \partial x \partial y} = (36x^2y^2+48x^3y^5+9x^4y^8)e^{xy^3}$