Math 201 Quiz 3 (B)

Q.1: Describe the traces of the quadratic surface $4x^2 + 25y^2 + z^2 = 100$ in the planes $x = k,\ y = k$, and $z = k$. Write the interval for each k.

Sol: For $x = k$, $4k^2 + 25y^2 + z^2 = 100 \Rightarrow 25y^2 + z^2 = 100 - 4k^2$ is a ellipse in a plane parallel to the $yz - plane$ and $|k| \le 5$.

For $y = k$, $4x^2 + 25k^2 + z^2 = 100 \Rightarrow 4x^2 + z^2 = 100 - 25k^2$ is a ellipse in a plane parallel to the $xz - plane$ and $|k| \le 2$.

For $z = k$, $4x^2 + 25y^2 + k^2 = 100 \Rightarrow 4x^2 + 25y^2 = 100 - k^2$ is a ellipse in a plane parallel to the $xy - plane$ and $|k| \le 10$.

Q.2: Change the spherical coordinates $\left(8, \frac{\pi}{3}, \frac{\pi}{4}\right)$ to cylindrical coordinates.

Sol: Given $\rho = 8$, $\theta = \frac{\pi}{3}$, and $\phi = \frac{\pi}{4}$.

For cylindrical coordinates $r = \rho \sin(\phi) = 8 \sin\left(\frac{\pi}{4}\right) = 8\frac{\sqrt{2}}{2} = 4\sqrt{2}$, $z = \rho\cos(\phi) \Rightarrow 8\cos\left(\frac{\pi}{4}\right) = 8\frac{\sqrt{2}}{2} = 4\sqrt{2}$.

Thus the cylindrical coordinates are $\left(4\sqrt{2}, \frac{\pi}{3}, 4\sqrt{2}\right)$.

Q.2: Identify the surface given by $\rho^2\left(4\sin^2(\phi) + 9\cos^2(\phi)\right) = 36$.

Sol: $\rho^2\left(4\sin^2(\phi) + 9\cos^2(\phi)\right) = 36$

$\Rightarrow 4\rho^2\sin^2(\phi) + 9\rho^2\cos^2(\phi) = 36$

$\Rightarrow 4r^2 + 9z^2 = 36 \Rightarrow 4x^2 + 4y^2 + 9z^2 = 36$

$\Rightarrow \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{4} = 1$.It is Ellipsoid.