Q.1: Describe the traces of the quadratic surface $4x^2 + 25y^2 + z^2 = 100$ in the planes x = k, y = k, and z = k. Write the interval for each k.

Sol: For x = k, $4k^2 + 25y^2 + z^2 = 100 \Rightarrow 25y^2 + z^2 = 100 - 4k^2$ is a ellipse in a plane parallel to the yz - plane and $|k| \le 5$.

For y = k, $4x^2 + 25k^2 + z^2 = 100 \Rightarrow 4x^2 + z^2 = 100 - 25k^2$ is a ellipse in a plane parallel to the xz - plane and $|k| \le 2$.

For z = k, $4x^2 + 25y^2 + k^2 = 100 \Rightarrow 4x^2 + 25y^2 = 100 - k^2$ is a ellipse in a plane parallel to the xy - plane and $|k| \le 10$.

Q.2: Change the spherical coordinates $\left(8, \frac{\pi}{3}, \frac{\pi}{4}\right)$ to cylindrical coordinates.

Sol: Given $\rho = 8$, $\theta = \frac{\pi}{3}$, and $\phi = \frac{\pi}{4}$.

For cylindrical coordinates $r = \rho \sin(\phi) = 8 \sin\left(\frac{\pi}{4}\right) = 8\frac{\sqrt{2}}{2} = 4\sqrt{2}, \ z = \rho \cos(\phi) \Rightarrow 8\cos\left(\frac{\pi}{4}\right) = 8\frac{\sqrt{2}}{2} = 4\sqrt{2}.$ Thus the ordinates approximates are $\left(4\sqrt{2}, \frac{\pi}{4}, 4\sqrt{2}\right)$

Thus the cylindrical coordinates are $\left(4\sqrt{2}, \frac{\pi}{3}, 4\sqrt{2}\right)$.

Q.2: Identify the surface given by $\rho^2 \left(4 \sin^2(\phi) + 9 \cos^2(\phi)\right) = 36.$

Sol: $\rho^2 \left(4\sin^2(\phi) + 9\cos^2(\phi) \right) = 36$ $\Rightarrow 4\rho^2 \sin^2(\phi) + 9\rho^2 \cos^2(\phi) = 36$ $\Rightarrow 4r^2 + 9z^2 = 36 \Rightarrow 4x^2 + 4y^2 + 9z^2 = 36$ $\Rightarrow \frac{x^2}{9} + \frac{y^2}{9} + \frac{z^2}{4} = 1.$ It is Ellipsoid.