Solution Math 201-092 Quiz 3 (B)

Q.1: Find the limit if it exist $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+9}-3}$.

Sol: $$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+9}-3} = \lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+9}-3}\cdot\frac{\sqrt{x^2+y^2+9}+3}{\sqrt{x^2+y^2+9}+3}$$

$$\lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\left(\sqrt{x^2+y^2+9}+3\right)}{x^2+y^2+9-9} = \lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2+9}+3 = 6.$$

Q.2: Use implicit differentiation to fine the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for $\cos(xyz) = 4x+3y+z$.

Sol: Implicit differentiation to find $\dfrac{\partial z}{\partial x}$,

$$-\sin(xyz)\left(yz+xy\frac{\partial z}{\partial x}\right) = 4+0+\frac{\partial z}{\partial x} \Rightarrow (-xy\sin(xyz)-1)\frac{\partial z}{\partial x} = 4+yz\ \sin(xyz)$$

$$\frac{\partial z}{\partial x} = \frac{4+yz\ \sin(xyz)}{-xy\sin(xyz)-1} = -\frac{4+yz\ \sin(xyz)}{xy\sin(xyz)+1}$$

We can also use chain rule to find these derivatives. Let $F(x,y,z) = \cos(xyz)-4x-3y-z=0$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{-\sin(xyz)\,xz-3}{-\sin(xyz)\,xy-1} = -\frac{xz\sin(xyz)+3}{xy\sin(xyz)+1}.$$

Q.3: Find equation of tangent plane to the surface $z=\sqrt{7-x^2-2y^2}$ at the point $(1,-1,2)$. Using linearization $L(x,y)$ of $f(x,y)=\sqrt{7-x^2-2y^2}$ at $(1,-1)$, approximate $f(0.99,-1.01)$.

Sol: $f_x = \dfrac{-2x}{2\sqrt{7-x^2-2y^2}}$ and $f_y = \dfrac{-4y}{2\sqrt{7-x^2-2y^2}}$

$f_x(1,-1) = -\dfrac{1}{2}$ and $f_y(1,-1) = 1$

Equation of tangent plane is $z-2 = -\dfrac{1}{2}(x-1)+(y+1) \Rightarrow z = -\dfrac{1}{2}x+y+\dfrac{7}{2}$.

Linearization of $f(x,y)$ at $(1,-1)$ is $L(x,y) = -\dfrac{1}{2}x+y+\dfrac{7}{2}$.

$f(1.01,-0.99) = -\dfrac{1}{2}(0.99)+(-1.01)+\dfrac{7}{2} = 1.995.$