Solution Math 201 Quiz 3 (B)

Q.1: Find the limit $\lim_{(x,y)\to(0,0)} \frac{3x^2+\sin^2 y}{2x^2+3y^2}$ if exist, or show that limit does not exist.

Sol: Let $(x,y)\to(0,0)$ through $y-axis$, that is $x=0$ and $y\to 0$, then

$$\lim_{(x,y)\to(0,0)} \frac{3x^2+\sin^2 y}{2x^2+3y^2} = \lim_{y\to 0}\frac{\sin^2 y}{3y^2} = \frac{1}{3}\lim_{y\to 0}\left(\frac{\sin y}{y}\right)^2 = \frac{1}{3}.$$

Now let $(x,y)\to(0,0)$ through $x-axis$, that is $y=0$ and $x\to 0$, then

$$\lim_{(x,y)\to(0,0)} \frac{3x^2+\sin^2 y}{2x^2+3y^2} = \lim_{x\to 0}\frac{3x^2}{2x^2} = \frac{3}{2}.$$

Since these two limits are not same, therefore limit does not exist.

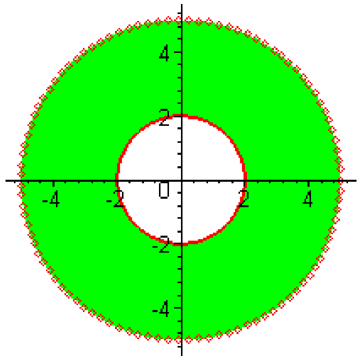
Q.2: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the function $F(x,y,z) = 2x^2+3y^2+4z^3-5xy+2xz-3yz+9$.

Sol: $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{4x-5y+2z}{12z^2+2x-3y}$ and $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{6y-5x-3z}{12z^2+2x-3y}$.

Q.3: Find and sketch the domain of the function $f(x,y) = \sqrt{x^2+y^2-4} + \ln\left(25-x^2-y^2\right)$. Write the domain in words.

Sol: $x^2+y^2-4 \ge 0 \Rightarrow x^2+y^2 \ge 4$ and $25-x^2-y^2 > 0 \Rightarrow x^2+y^2 < 25$.

The domain is inside the open circle of radius 5 and outside the circle of radius 2.



Q4: Show that $u(x,t) = \sin(x-at) + \ln(x+at)$ is a solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2}$.

Sol: $\frac{\partial u}{\partial x} = \cos(x-at) + \frac{1}{x+at}$ and $\frac{\partial^2 u}{\partial x^2} = -\sin(x-at) + \frac{-1}{(x+at)^2}$

$\frac{\partial u}{\partial t} = -a\cos(x-at) + \frac{a}{x+at}$ and $\frac{\partial^2 u}{\partial t^2} = -a^2\sin(x-at) + \frac{-a^2}{(x+at)^2} = a^2\frac{\partial^2 u}{\partial x^2}$.

Q.5: Show that $f(x,y) = 2xe^{xy}$ is differentiable at the point $P(2,0)$ and find the linearization $L(x,y)$ of $f(x,y)$ at the point $P(2,0)$. Use $L(x,y)$ to approximate $f(2.1,-0.1)$.

Sol: $\frac{\partial f}{\partial x} = 2e^{xy} + 2xye^{xy}$ and $\frac{\partial f}{\partial y} = 2x^2e^{xy}$

$\frac{\partial f(2,0)}{\partial x} = 2$ and $\frac{\partial f(2,0)}{\partial y} = 8$. Since both derivatives exist at $P(2,0)$, therefore f is differentiable at $P(2,0)$.

$L(x,y) = f(a,b) + \frac{\partial f(a,b)}{\partial x}(x-a) + \frac{\partial f(a,b)}{\partial x}(y-b) = 4 + 2(x-2) + 8(y-0) = 2x+8y$

$L(1.1,-0.1) = 2(2.1) + 8(-0.1) = 4.2 - 0.8 = 3.4.$

Q.6: If $z = f(x,y) = 2x^2+xy-3y^2$, find the differential dz. If x changes from 1 to 1.1 and y changes from 2 to 2.05, compute the value of dz.

Sol: $dz = f_x dx + f_y dy = (4x+y)\,dx + (x-6y)\,dy$.

With $x=1,\ y=2$ and $dx = \Delta x = 0.1,\ dy = \Delta y = 0.05$,

$dz = (4+2)(0.1) + (1-12)(0.05) = 0.6 - 0.55 = 0.05.$