

Q.1: Describe the traces of the quadratic surface $9x^2 + 4y^2 + 36z^2 = 36$ in the planes $x = k$, $y = k$, and $z = k$. Write the interval for each k .

Sol: For $x = k$, $9k^2 + 4y^2 + 36z^2 = 36 \Rightarrow 4y^2 + 36z^2 = 36 - 9k^2$ is an ellipse in a plane parallel to the yz -plane and $|k| \leq 2$.

For $y = k$, $9x^2 + 4k^2 + 36z^2 = 36 \Rightarrow 9x^2 + 36z^2 = 36 - 4k^2$ is an ellipse in a plane parallel to the xz -plane and $|k| \leq 3$.

For $z = k$, $9x^2 + 4y^2 + 36k^2 = 36 \Rightarrow 9x^2 + 4y^2 = 36 - 36k^2$ is an ellipse in a plane parallel to the xy -plane and $|k| \leq 1$.

Q.2: Change the spherical coordinates $\left(4, \frac{\pi}{6}, \frac{\pi}{2}\right)$ to cylindrical coordinates.

Sol: Given $\rho = 4$, $\theta = \frac{\pi}{6}$, and $\phi = \frac{\pi}{2}$.

For cylindrical coordinates $r = \rho \sin(\phi) = 4 \sin\left(\frac{\pi}{2}\right) = 4$, $z = \rho \cos(\phi) \Rightarrow 4 \cos\left(\frac{\pi}{2}\right) = 0$.

Thus the cylindrical coordinates are $\left(4, \frac{\pi}{6}, 0\right)$.

Q.2: Identify the surface given by $\rho^2 (9 \sin^2(\phi) - 4 \cos^2(\phi)) = 36$.

Sol: $\rho^2 (9 \sin^2(\phi) - 4 \cos^2(\phi)) = 36$
 $\Rightarrow 9\rho^2 \sin^2(\phi) - 4\rho^2 \cos^2(\phi) = 36$
 $\Rightarrow 9r^2 - 4z^2 = 36 \Rightarrow 9x^2 + 9y^2 - 4z^2 = 36$
 $\Rightarrow \frac{x^2}{4} + \frac{y^2}{4} - \frac{z^2}{9} = 1$. It is a Hyperboloid of one sheet.