Solution Math 201-092 Quiz 3 (A)

Q.1: Find the limit if it exist $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$.

Sol: $$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = \lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} \cdot \frac{\sqrt{x^2+y^2+1}+1}{\sqrt{x^2+y^2+1}+1}$$

$$\lim_{(x,y)\to(0,0)} \frac{(x^2+y^2)\left(\sqrt{x^2+y^2+1}+1\right)}{x^2+y^2+1-1} = \lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2+1}+1 = 2.$$

Q.2: Use implicit differentiation to fine the partial derivatives $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for $\sin(xyz) = x+2y+3z$.

Sol: Implicit differentiation to find $\dfrac{\partial z}{\partial x}$,

$$\cos(xyz)\left(yz + xy\frac{\partial z}{\partial x}\right) = 1+0+3\frac{\partial z}{\partial x} \Rightarrow (xy\cos(xyz)-3)\frac{\partial z}{\partial x} = 1 - yz\ \cos(xyz)$$

$$\frac{\partial z}{\partial x} = \frac{1-yz\ \cos(xyz)}{xy\cos(xyz)-3}.$$

We can also use chain rule to find these derivatives. Let $F(x,y,z) = \sin(xyz) - x - 2y - 3z = 0$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{\cos(xyz)\,xz-2}{\cos(xyz)\,xy-3} = \frac{2-\cos(xyz)\,xz}{\cos(xyz)\,xy-3}.$$

Q.3: Find equation of tangent plane to the surface $z = \sqrt{4-x^2-2y^2}$ at the point $(1,-1,1)$. Using linearization $L(x,y)$ of $f(x,y) = \sqrt{4-x^2-2y^2}$ at $(1,-1)$, approximate $f(1.01,-0.99)$.

Sol: $f_x = \dfrac{-2x}{2\sqrt{4-x^2-2y^2}}$ and $f_y = \dfrac{-4y}{2\sqrt{4-x^2-2y^2}}$

$f_x(1,-1) = -1$ and $f_y(1,-1) = 2$

Equation of tangent plane is $z-1 = -1(x-1) + 2(y+1) \Rightarrow z = -x+2y+4$.

Linearization of $f(x,y)$ at $(1,-1)$ is $L(x,y) = -x+2y+4$.

$f(1.01,-0.99) = -1.01 - 2(0.99) + 4 = 1.01.$