Math 201 Quiz 2 (Revised) (B)

Q.1: Find parametric equations of the line of intersection of the planes $3x + 2y - 3z = 5$, $2x - 3y + 4z = 3$.

Sol: Normal to these planes are $\vec{n}_1 = \langle 3, 2, -3 \rangle$ and $\vec{n}_2 = \langle 2, -3, 4 \rangle$. Since $\vec{n}_1 \nparallel \vec{n}_2$, therefore these two planes intersect in a line.

Solving these two equations,

$$\begin{array}{c} 9x + 6y - 9z = 15 \\ 4x - 6y + 8z = 6 \\ \hline 13x + 0y \;-\; z = 21 \end{array}$$

we get $z = 13x - 21$ and $2y = -3x + 3z + 5 = -3x + 39x - 63 + 5 = 36x - 58$, or $y = 18x - 29$.

Let $x = 0$, then $y = -29$ and $z = -21$. Now let $x = 1$, then $y = -11$ and $z = -8$.

So $(0, -29, -21)$ and $(1, -11, -8)$ are two points on the line of intersection and $\vec{v} = \langle -1, -18, -13 \rangle$ is a vector parallel to the line. Therefore parametric equations of the line are:

$$\begin{array}{c} x = -t \\ y = -29 - 18t \\ z = -21 - 13t \end{array} \quad \text{or} \quad \begin{array}{c} x = 1 - s \\ y = -11 - 18s \\ z = -8 - 13s \end{array}$$

Q.2: Find the value of k such that the plane passes through the line of intersection of the planes $3x + 2y - 3z = 5$, $2x - 3y + 4z = 3$ is parallel to the plane $2x + 3ky - 4z = 5$.

Sol: Required plane is parallel to the plane $2x + 3ky - 4z = 5$ and therefore have same normal vectors $\vec{n} = \langle 2, 3k, -4 \rangle$.

Since required plane is passing through the line with parallel vector $\vec{v} = \langle -1, -18, -13 \rangle$, therefore $\vec{v} \cdot \vec{n}$ should be equal to zero.

But $\vec{n} \cdot \vec{v} = -2 - 54k + 52 = 0$

$54k = 50$ or $k = \dfrac{50}{54} = \dfrac{25}{27}$.

Q.3: Reduce the equation to standard form and classify the surface, $x^2 - 2y^2 + z^2 - 4x + 4y + 4z + 4 = 0$.

Sol: $x^2 - 4x - 2y^2 + 4y + z^2 + 4z = -4$

$(x^2 - 4x + 4) - 2(y^2 - 2y + 1) + (z^2 + 4z + 4) = -4 + 4 - 2 + 4$

$(x - 2)^2 - 2(y - 1)^2 + (z + 2)^2 = 2$

This is hyperboloid of one sheet.

Q.4: Identify the surface whose equation if given by $\rho^2 \left(7 \sin^2 \phi \cos(2\theta) - 6 \cos^2 \phi\right) = 5$.

Sol: $7\rho^2 \sin^2 \phi \left(\cos^2 \theta - \sin^2 \theta\right) - 6\rho^2 \cos^2 \phi = 5$

$7\rho^2 \sin^2 \phi \cos^2 \theta - 7\rho^2 \sin^2 \phi \sin^2 \theta - 6\rho^2 \cos^2 \phi = 5$

$7x^2 - 7y^2 - 6z^2 = 5.$ Hyperboloid of two sheets.