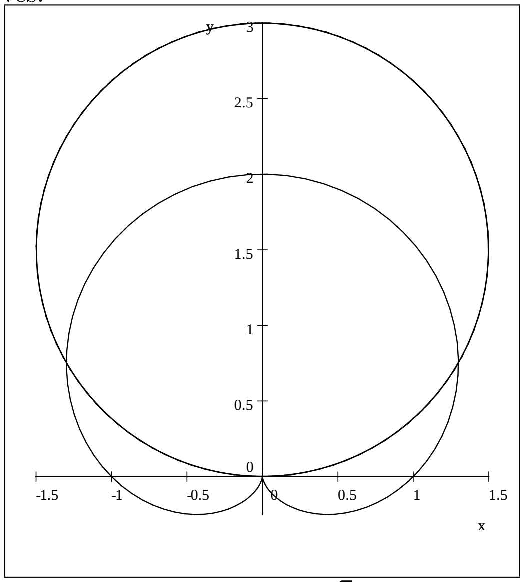
Soluttion Math 201 Quiz 2

(B)

Q.1: Graphs of $r = 1 + \sin(\theta)$ and $r = 3\sin(\theta)$ are shown in the figure. Find the area that lies inside both curves.



$1 + \sin(\theta) = 3\sin(\theta)$ for $\theta = \dfrac{\pi}{6}$ and Area

$$A = 2\left(\frac{1}{2}\int_0^{\frac{\pi}{6}} 9\sin^2(\theta)d\theta + \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1+\sin(\theta))^2\, d\theta\right)$$

$$= 9\int_0^{\frac{\pi}{6}} \sin^2(\theta)\, d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(1 + 2\sin(\theta) + \sin^2(\theta)\right) d\theta$$

$$= 9\int_0^{\frac{\pi}{6}} \frac{1-\cos(2\theta)}{2} d\theta + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(1 + 2\sin(\theta) + \frac{1-\cos(2\theta)}{2}\right) d\theta$$

$$= 9\left(\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right)\Bigg|_0^{\frac{\pi}{6}} + \left(\theta - 2\cos(\theta) + \frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right)\Bigg|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = \frac{5}{4}\pi$$

Q.2: Find equation of the sphere with center $(4, 2, 3)$ and radius equal to 5. Write name and equations of its intersections with coordinate planes.

Equation of the sphere $(x-4)^2 + (y-2)^2 + (z-3)^2 = 25$

Equation of the intersection with $xy-$plane, put $z = 0$, $(x-4)^2 + (y-2)^2 + (0-3)^2 = 25$
$\implies (x-4)^2 + (y-2)^2 = 16$, a circle in $xy-$plane

Equation of the intersection with $yz-$plane, put $x = 0$, $(0-4)^2 + (y-2)^2 + (z-3)^2 = 25$
$\implies (y-2)^2 + (z-3)^2 = 9$, a circle in $yz-$plane

Equation of the intersection with $xz-$plane, put $y = 0$, $(x-4)^2 + (0-2)^2 + (z-3)^2 = 25$
$\implies (x-4)^2 + (z-3)^2 = 21$, a circle in $xz-$plane