Solution Math 201 Quiz 2 (B)

Q.1: Find parametric equations of the line of intersection of the planes $x+y+z=1,\ \ x-2y+3z=1$.
Sol: Normal to these planes are $\vec{n}_1=\langle 1,1,1\rangle$ and $\vec{n}_2=\langle 1,-2,3\rangle$. Since $\vec{n}_1 \nparallel \vec{n}_2$, therefore these two planes intersect in a line.

Solving these two equations,

$$\begin{array}{c} x+y+z=1 \\ x-2y+3z=1 \\ \hline 3y\ -\ 2z=0 \end{array}$$

we get $z=\dfrac{3}{2}y$ and $x=1-y-z=1-y-\dfrac{3}{2}y=1-\dfrac{5}{2}y$.

Let $y=0$, then $x=1$ and $z=0$. Now let $y=2$, then $x=-4$ and $z=3$.

So $(1,0,0)$ and $(-4,2,3)$ are two points on the line of intersection and $\vec{v}=\langle -5,2,3\rangle$ is a vector parallel to the line. Therefore parametric equations of the line are:

$$\begin{array}{rcl} x=-4-5t & & x=1-5s \\ y=2+2t & \text{or} & y=0+2s \\ z=3+3t & & z=0+3s \end{array}$$

Q.2: Find equation of the plane that passes through the line of intersection of the planes $x+y-z=2,\ \ 3x-4y+5z=6$ and is parallel to the plane $x+3y-2z=5$.
Sol: Required plane is parallel to the plane $x+3y-2z=5$ and therefore have same normal vectors $\vec{n}=\langle 1,3,-2\rangle$.

Since required plane is passing through the line with parallel vector $\vec{v}=\langle -5,2,3\rangle$, therefore $\vec{v}\cdot\vec{n}$ should be equal to zero.

But $\vec{n}\cdot\vec{v}=-5+6-6=-5\neq 0$. So there is no plane which is passing through the line of intersection of the planes $x+y-z=2,\ \ 3x-4y+5z=6$ and is parallel to the plane $x+3y-2z=5$.

Q.3: Reduce the equation to standard form and classify the surface, $x^2+y^2-z^2-6x+4y+8z+5=0$.
Sol: $x^2-6x+y^2+4y-z^2+8z=-5$

$x^2-6x+9+y^2+4y+4-z^2+8z-16=-5+9+4-16$

$(x-3)^2+(y+2)^2-(z-4)^2=-8$ or $-(x-3)^2-(y+2)^2+(z-4)^2=8$

This is hyperboloid of two sheets.

Q.4: Identify the surface whose equation if given by $\rho^2\left(-3\sin^2\phi\cos(2\theta)+2\cos^2\phi\right)=1$.
Sol: $-3\rho^2\sin^2\phi\left(\cos^2\theta-\sin^2\theta\right)+2\rho^2\cos^2\phi=1$

$-3\rho^2\sin^2\phi\cos^2\theta+3\rho^2\sin^2\phi\sin^2\theta+2\rho^2\cos^2\phi=1$

$-3x^2+3y^2+2z^2=1$

Hyperboloid of one sheet.