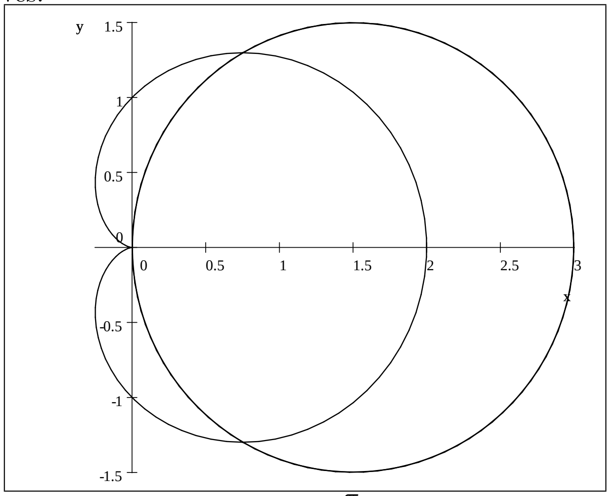
Q.1: Graphs of $r = 1 + \cos(\theta)$ and $r = 3\cos(\theta)$ are shown in the figure. Find the area that lies inside both curves.



$1 + \cos(\theta) = 3\cos(\theta)$ for $\theta = \dfrac{\pi}{3}$ and Area

$$A = 2\left(\frac{1}{2}\int_0^{\frac{\pi}{3}} (1+\cos(\theta))^2\, d\theta + \frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9\cos^2(\theta) d\theta\right)$$

$$= \int_0^{\frac{\pi}{3}} \left(1 + 2\cos(\theta) + \cos^2(\theta)\right) d\theta + 9\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2(\theta) d\theta$$

$$= \int_0^{\frac{\pi}{3}} \left(1 + 2\cos(\theta) + \frac{1+\cos(2\theta)}{2}\right) d\theta + 9\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1+\cos(2\theta)}{2} d\theta$$

$$= \left(\theta + 2\sin(\theta) + \frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right)\Bigg|_0^{\frac{\pi}{3}} + 9\left(\frac{\theta}{2} + \frac{\sin(2\theta)}{4}\right)\Bigg|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \frac{5}{4}\pi$$

Q.2: Find equation of the sphere with center $(-4, 3, -2)$ and radius equal to 5. Write name and equations of its intersections with coordinate planes.

Equation of the sphere $(x+4)^2 + (y-3)^2 + (z+2)^2 = 25$

Equation of the intersection with $xy-$plane, put $z = 0$, $(x+4)^2 + (y-3)^2 + (0+2)^2 = 25$
$\Longrightarrow$ $(x+4)^2 + (y-3)^2 = 21$, a circle in $xy-$plane

Equation of the intersection with $yz-$plane, put $x = 0$, $(0+4)^2 + (y-3)^2 + (z+2)^2 = 25$
$\Longrightarrow$ $(y-3)^2 + (z+2)^2 = 9$, a circle in $yz-$plane

Equation of the intersection with $xz-$plane, put $y = 0$, $(x+4)^2 + (0-3)^2 + (z+2)^2 = 25$
$\Longrightarrow$ $(x+4)^2 + (z+2)^2 = 16$, a circle in $xz-$plane