Solution Math 201 Quiz 2 (A)

Q.1: Find parametric equations of the line of intersection of the planes $x+y-z=2,\ 3x-4y+5z=6$.
Sol: Normal to these planes are $\vec{n}_1=\langle 1,1,-1\rangle$ and $\vec{n}_2=\langle 3,-4,5\rangle$. Since $\vec{n}_1 \nparallel \vec{n}_2$, therefore these two planes intersect in a line.

Solving these two equations,

$$\begin{array}{r} 4x+4y-4z=8 \\ 3x-4y+5z=6 \\ \hline 7x+0y\ +\ z=14 \end{array}$$

we get $z=14-7x$ and $y=2-x+z=2-x+14-7x=16-8x$.

Let $x=1$, then $y=8$ and $z=7$. Now let $x=2$, then $y=0$ and $z=0$.

So $(1,8,7)$ and $(2,0,0)$ are two points on the line of intersection and $\vec{v}=\langle -1,8,7\rangle$ is a vector parallel to the line. Therefore parametric equations of the line are:

$$\begin{array}{ccc} x=1-t & & x=2-s \\ y=8+8t & \text{or} & y=0+8s \\ z=7+7t & & z=0+7s \end{array}$$

Q.2: Find equation of the plane that passes tgrough the line of intersection of the planes $x+y-z=2,\ \ 3x-4y+5z=6$ and is parallel to the plane $2x+y-z=4$.
Sol: Required plane is parallel to the plane $2x+y-z=4$ and therefore have same normal vectors $\vec{n}=\langle 2,1,-1\rangle$.

Since required plane is passing through the line with parallel vector $\vec{v}=\langle -1,8,7\rangle$, therefore $\vec{v}\cdot\vec{n}$ should be equal to zero.

But $\vec{n}\cdot\vec{v}=-2+8-7=-1\neq 0$. So there is no plane which is passing through the line of intersection of the planes $x+y-z=2,\ \ 3x-4y+5z=6$ and is parallel to the plane $2x+y-z=4$.

Q.3: Reduce the equation to standard form and classify the surface, $x^2-y^2+z^2-2x+2y+4z+2=0$.
Sol: $x^2-2x-y^2+2y+z^2+4z=-2$

$x^2-2x+1-y^2+2y-1+z^2+4z+4=-2+1-1+4$

$(x-1)^2-(y-1)^2+(z+2)^2=2$

This is hyperboloid of one sheet.

Q.4: Identify the surface whose equation if given by $\rho^2\left(2\sin^2\phi\cos(2\theta)-3\cos^2\phi\right)=1$.
Sol: $2\rho^2\sin^2\phi\left(\cos^2\theta-\sin^2\theta\right)-3\rho^2\cos^2\phi=1$

$2\rho^2\sin^2\phi\cos^2\theta-2\rho^2\sin^2\phi\sin^2\theta-3\rho^2\cos^2\phi=1$

$2x^2-2y^2-3z^2=1$

Hyperboloid of two sheets.