Solution Math 201 Quiz 1 (A)

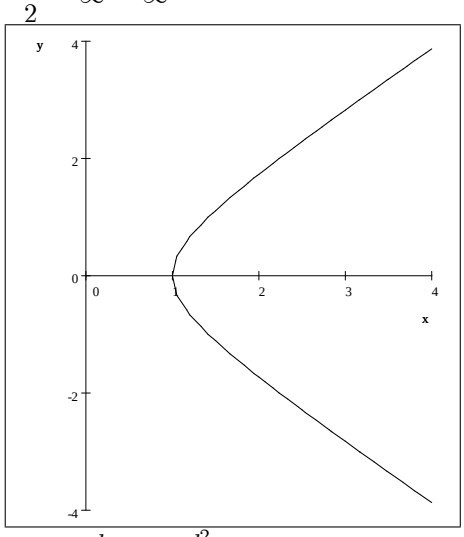
Q.1: Elliminate the parameter t from the parametric equations $x = \sec(t),\ y = \tan(t)$ to find a cartesian equation. Sketch the graph and mark the direction in which the curve is traced for $-\dfrac{\pi}{2} < t < \dfrac{\pi}{2}$.

Sol: $x^2 = \sec^2 t,\ y^2 = \tan^2 t$

$1 + y^2 = 1 + \tan^2 t = \sec^2 t = x^2$

$x^2 - y^2 = 1$, a hyperbola.

t	x	y
$-\dfrac{\pi}{2}$	∞	$-\infty$
0	1	0
$\dfrac{\pi}{2}$	∞	∞



Q.2: Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for the parametric equations given in Question 1.

Sol: $$\frac{dy}{dx} = \frac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t} = \frac{1}{\cos t}\frac{\cos t}{\sin t} = \csc t.$$

$$\frac{d^2y}{dx^2} = \frac{\dfrac{d}{dt}\left(\dfrac{dy}{dx}\right)}{\dfrac{dx}{dt}} = \frac{\dfrac{d}{dt}(\csc t)}{\sec t \tan t} = \frac{-\csc t \cot t}{\sec t \tan t} = -\cot^3 t.$$

Q.3: Sketch the graph of the polar equation $r = 3 - 2\sin(\theta)$.