Math 102 Solution Quiz 6 (B)

Q.1: Determine whether the integral converges or diverges $\int\limits_1^\infty \frac{1}{x^2+5x+6}dx$.

Sol: $\int\limits_1^\infty \frac{1}{x^2+5x+6}dx = \lim\limits_{t\to\infty}\int\limits_1^t \frac{1}{x^2+5x+6}dx = \lim\limits_{t\to\infty}\int\limits_1^t \left(\frac{1}{x+2} - \frac{1}{x+3}\right)dx$

$$= \lim_{t\to\infty}\left[\ln(x+2) - \ln(x+3)\right]\Big|_1^t$$

$$= \lim_{t\to\infty}\left[\ln\left(\frac{x+2}{x+3}\right)\right]\Bigg|_1^t$$

$$= \lim_{t\to\infty}\left[\ln\left(\frac{t+2}{t+3}\right) - \ln\left(\frac{1+2}{1+3}\right)\right]$$

$$= \ln(1) - \ln\left(\frac{3}{4}\right) = \ln\left(\frac{4}{3}\right).$$

Thus the integral converges.

Q.2: Determine whether the sequence converges or diverges $a_n = \frac{2^n}{3^{n+1}}$.

Sol: $\lim\limits_{t\to\infty} a_n = \lim\limits_{t\to\infty}\frac{2^n}{3\,3^n} = \frac{1}{3}\lim\limits_{t\to\infty}\left(\frac{2}{3}\right)^n = \frac{1}{3}(0) = 0.$

Thus the sequence converges.

Q.3: Determine whether the series converges or diverges $\sum\limits_{n=1}^\infty \frac{(n+1)^2}{n(n+5)}$.

Sol: $\lim\limits_{t\to\infty} a_n = \lim\limits_{t\to\infty}\frac{(n+1)^2}{n(n+5)} = \lim\limits_{t\to\infty}\frac{n^2+2n+1}{n^2+5n} = \lim\limits_{t\to\infty}\frac{1+\frac{2}{n}+\frac{1}{n^2}}{1+\frac{5}{n}} = \frac{1}{1} = 1 \neq 0.$

Thus the series diverges by **Divergence Theorem**.

Q.4: Determine whether the series converges or diverges $\sum\limits_{n=1}^\infty \frac{1}{(n^2+6n+10)}$.

Sol: Let $f(x) = \frac{1}{x^2+6x+10} = \frac{1}{(x+3)^2+1}$

$\int\limits_1^\infty \frac{1}{(x+3)^2+1}dx = \lim\limits_{t\to\infty}\int\limits_1^t \frac{1}{(x+3)^2+1}dx = \lim\limits_{t\to\infty}\left(\tan^{-1}(x+3)\right)\Big|_1^t$

$$= \tan^{-1}(\infty) - \tan^{-1}(4) = \frac{\pi}{2} - \tan^{-1}(4).$$

The integral converges, therefore the series also converges by **Integral Test**.