Math 102 Solution Quiz 6 (A)

Q.1: Determine whether the integral converges or diverges $\int_1^\infty \frac{1}{x^2+3x+2}dx$.

Sol: $$\int_1^\infty \frac{1}{x^2+3x+2}dx = \lim_{t\to\infty}\int_1^t \frac{1}{x^2+3x+2}dx = \lim_{t\to\infty}\int_1^t \left(\frac{1}{x+1}-\frac{1}{x+2}\right)dx$$
$$= \lim_{t\to\infty} \left[\ln(x+1)-\ln(x+2)\right]\Big|_1^t$$
$$= \lim_{t\to\infty} \left[\ln\left(\frac{x+1}{x+2}\right)\right]\Big|_1^t$$
$$= \lim_{t\to\infty} \left[\ln\left(\frac{t+1}{t+2}\right)-\ln\left(\frac{1+1}{1+2}\right)\right]$$
$$= \ln(1)-\ln\left(\frac{2}{3}\right) = \ln\left(\frac{3}{2}\right).$$

Thus the integral converges.

Q.2: Determine whether the sequence converges or diverges $a_n = \frac{3^n}{4^{n+2}}$.

Sol: $$\lim_{t\to\infty} a_n = \lim_{t\to\infty}\frac{3^n}{16\ 4^n} = \frac{1}{16}\lim_{t\to\infty}\left(\frac{3}{4}\right)^n = \frac{1}{16}(0) = 0.$$

Thus the sequence converges.

Q.3: Determine whether the series converges or diverges $\sum_{n=1}^\infty \frac{(n+2)^2}{n(n+1)}$.

Sol: $$\lim_{t\to\infty} a_n = \lim_{t\to\infty}\frac{(n+2)^2}{n(n+1)} = \lim_{t\to\infty}\frac{n^2+4n+4}{n^2+n} = \lim_{t\to\infty}\frac{1+\frac{4}{n}+\frac{1}{n^2}}{1+\frac{1}{n}} = \frac{1}{1} = 1 \neq 0.$$

Thus the series diverges by **Divergence Theorem**.

Q.4: Determine whether the series converges or diverges $\sum_{n=1}^\infty \frac{1}{(n^2+4n+5)}$.

Sol: Let $f(x) = \frac{1}{x^2+4x+5} = \frac{1}{(x+2)^2+1}$

$$\int_1^\infty \frac{1}{(x+2)^2+1}dx = \lim_{t\to\infty}\int_1^t \frac{1}{(x+2)^2+1}dx = \lim_{t\to\infty}\left(\tan^{-1}(x+2)\right)\Big|_1^t$$
$$= \tan^{-1}(\infty)-\tan^{-1}(3) = \frac{\pi}{2}-\tan^{-1}(3).$$

The integral converges, therefore the series also converges by **Integral Test**.