

Q.1: Determine whether the integral converges or diverges $\int_1^{\infty} \frac{1}{x^2 + 3x + 2} dx$.

$$\begin{aligned} \text{Sol: } \int_1^{\infty} \frac{1}{x^2 + 3x + 2} dx &= \lim_{t \rightarrow \infty} \int_1^t \frac{1}{x^2 + 3x + 2} dx = \lim_{t \rightarrow \infty} \int_1^t \left(\frac{1}{x+1} - \frac{1}{x+2} \right) dx \\ &= \lim_{t \rightarrow \infty} [\ln(x+1) - \ln(x+2)]_1^t \\ &= \lim_{t \rightarrow \infty} \left[\ln \left(\frac{x+1}{x+2} \right) \right]_1^t \\ &= \lim_{t \rightarrow \infty} \left[\ln \left(\frac{t+1}{t+2} \right) - \ln \left(\frac{1+1}{1+2} \right) \right] \\ &= \ln(1) - \ln \left(\frac{2}{3} \right) = \ln \left(\frac{3}{2} \right). \end{aligned}$$

Thus the integral converges.

Q.2: Determine whether the sequence converges or diverges $a_n = \frac{3^n}{4^{n+2}}$.

$$\text{Sol: } \lim_{t \rightarrow \infty} a_n = \lim_{t \rightarrow \infty} \frac{3^n}{16 \cdot 4^n} = \frac{1}{16} \lim_{t \rightarrow \infty} \left(\frac{3}{4} \right)^n = \frac{1}{16} (0) = 0.$$

Thus the sequence converges.

Q.3: Determine whether the series converges or diverges $\sum_{n=1}^{\infty} \frac{(n+2)^2}{n(n+1)}$.

$$\text{Sol: } \lim_{t \rightarrow \infty} a_n = \lim_{t \rightarrow \infty} \frac{(n+2)^2}{n(n+1)} = \lim_{t \rightarrow \infty} \frac{n^2 + 4n + 4}{n^2 + n} = \lim_{t \rightarrow \infty} \frac{1 + \frac{4}{n} + \frac{1}{n^2}}{1 + \frac{1}{n}} = \frac{1}{1} = 1 \neq 0.$$

Thus the series diverges by **Divergence Theorem**.

Q.4: Determine whether the series converges or diverges $\sum_{n=1}^{\infty} \frac{1}{(n^2 + 4n + 5)}$.

$$\begin{aligned} \text{Sol: } \text{Let } f(x) &= \frac{1}{x^2 + 4x + 5} = \frac{1}{(x+2)^2 + 1} \\ \int_1^{\infty} \frac{1}{(x+2)^2 + 1} dx &= \lim_{t \rightarrow \infty} \int_1^t \frac{1}{(x+2)^2 + 1} dx = \lim_{t \rightarrow \infty} (\tan^{-1}(x+2)) \Big|_1^t \\ &= \tan^{-1}(\infty) - \tan^{-1}(3) = \frac{\pi}{2} - \tan^{-1}(3). \end{aligned}$$

The integral converges, therefore the series also converges by **Integral Test**.