Solution Math 102 Quiz 4 (A)

Q.1: Determine whether the series converges or diverges $\sum_{n=1}^{\infty} \frac{2n^2+3n}{4^n(3n+2)^2}$.

Sol: Here $a_n = \frac{2n^2+3n}{4^n(3n+2)^2}$, let $b_n = \frac{1}{4^n} = \left(\frac{1}{4}\right)^n$.

Then $\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2n^2+3n}{4^n(3n+2)^2}\frac{4^n}{1} = \lim_{n\to\infty} \frac{2n^2+3n}{(3n+2)^2} = \frac{2}{9} > 0.$

Since $\sum_{n=1}^{\infty} b_n$ converges (GS with $r = \frac{1}{4}$),

therefore $\sum_{n=1}^{\infty} a_n$ also converges by Limit Comparison Test.

Q.2: Determine whether the series converges or diverges $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\ln n}{n}$.

Sol: $a_n = \frac{(-1)^{n-1}\ln n}{n} = (-1)^{n-1} b_n$, where $b_n = \frac{\ln n}{n}$

$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{\ln n}{n} = \lim_{x\to\infty} \frac{\ln x}{x} \overset{H}{=} \lim_{x\to\infty} \frac{1}{x} = 0$

Let $f(x) = \frac{\ln x}{x}$, then $f'(x) = \frac{1-\ln x}{x^2} < 0$ for all $x > e$.

$f(x)$ decreasing $\Rightarrow b_n = \frac{\ln n}{n}$ is also decreasing.

Therefore the series converges by Alternating Series Test.

Q.3: Determine whether the series converges or diverges $\sum_{n=1}^{\infty} \frac{(n+1)!}{n^n}$.

Sol: $a_n = \frac{(n+1)!}{n^n}$, $a_{n+1} = \frac{(n+2)!}{(n+1)^{n+1}} = \frac{(n+2)(n+1)!}{(n+1)^n(n+1)} = \frac{(n+2)(n+1)!}{n^n\left(1+\frac{1}{n}\right)^n(n+1)}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+2)(n+1)!}{n^n\left(1+\frac{1}{n}\right)^n(n+1)} \cdot \frac{n^n}{(n+1)!} = \lim_{n\to\infty} \frac{n+2}{\left(1+\frac{1}{n}\right)^n(n+1)} = \frac{1}{e} < 1$

Therefore the series converges by the Ratio Test.