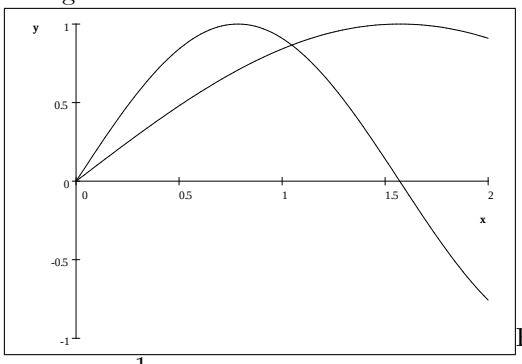
Solution of Math 102 Quiz 3 (B)

Name:.....Serial#:.....Sec #.....

Q.1: Find $f'(x)$ for $f(x) = \log_a\left(\tan^2 x\right)$.

Sol: $f(x) = 2\log_a(\tan x)$ and $f'(x) = 2\dfrac{\sec^2 x}{\ln a.\tan x}$

Q.2: Sketch the region bounded by the curves $y = \sin(x)$, $y = \sin(2x)$, $x = 0$, $x = \dfrac{\pi}{2}$. Also find the area of the region.



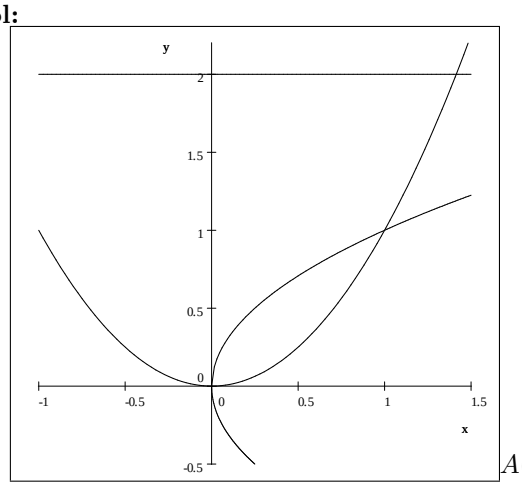
For the points of intersection, solve $\sin(2x) = \sin(x)$, Solution is: $x = 0, \dfrac{1}{3}\pi$.

$$A = \int_0^{\frac{\pi}{3}} (\sin(2x) - \sin(x))\,dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sin(x) - \sin(2x))\,dx = \left(-\frac{\cos(2x)}{2} + \cos(x)\right)\Big|_0^{\frac{\pi}{3}}$$

$$+ \left(-\cos(x) + \frac{\cos(2x)}{2}\right)\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} = \frac{1}{2}$$

Q.3: Find volume of the solid obtained by rotating the region bounded by $y = x^2$, $x = y^2$, about $y = 2$.

Sol:



$A(x) = \pi\left[\left(2 - x^2\right)^2 - \left(2 - \sqrt{x}\right)^2\right]$

$$V = \int_0^1 A(x)\,dx = \int_0^1 \pi\left[\left(2 - x^2\right)^2 - \left(2 - \sqrt{x}\right)^2\right]dx = \int_0^1 \pi\left(4 - 4x^2 + x^4 - 4 + 4\sqrt{x} - x\right)dx = \frac{31}{30}\pi$$