

# Solution of Math 102 Quiz 3

(A)

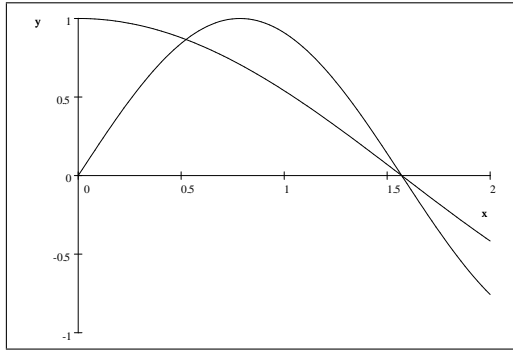
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**Q.1:** Find  $f'(x)$  for  $f(x) = \log_a(\sin^2 x)$ .

**Sol:**  $f(x) = 2 \log_a(\sin x)$  and  $f'(x) = 2 \frac{\cos x}{\ln a \cdot \sin x} = 2 \frac{\cot x}{\ln a}$

**Q.2:** Sketch the region bounded by the curves  $y = \cos(x)$ ,  $y = \sin(2x)$ ,  $x = 0$ ,  $x = \frac{\pi}{2}$ . Also find the area of the region.

**Sol:**



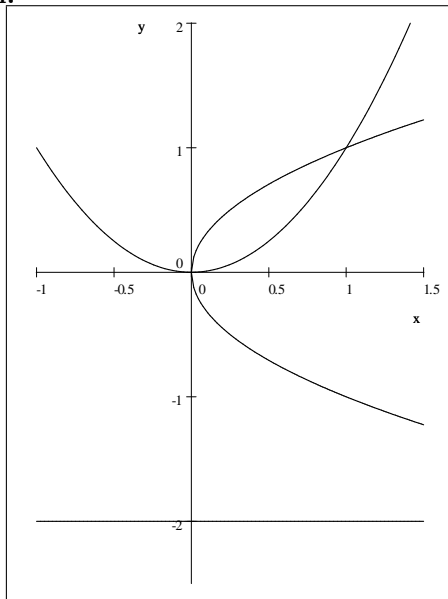
For the points of intersection, solve  $\sin(2x) = \cos(x)$ , Solution is:  $x = \frac{1}{2}\pi, \frac{1}{6}\pi$ .

$$A = \int_0^{\frac{\pi}{6}} (\cos(x) - \sin(2x)) dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin(2x) - \cos(x)) dx = \left( \sin(x) + \frac{\cos(2x)}{2} \right) \Big|_0^{\frac{\pi}{6}} + \left( -\frac{\cos(2x)}{2} - \sin(x) \right) \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} =$$

$\frac{1}{2}$

**Q.3:** Find volume of the solid obtained by rotating the region bounded by  $y = x^2$ ,  $x = y^2$ , about  $y = -2$ .

**Sol:**



$$A(x) = \pi \left[ (\sqrt{x} + 2)^2 - (x^2 + 2)^2 \right]$$

$$V = \int_0^1 A(x) dx = \int_0^1 \pi \left[ (\sqrt{x} + 2)^2 - (x^2 + 2)^2 \right] dx = \int_0^1 \pi (x + 4\sqrt{x} + 4 - x^4 - 4x^2 - 4) dx = \frac{49}{30} \pi.$$