Math 102 Quiz 1 (B)

Q.1: Estimate area under the graph of $y = 25 - x^2$ from $x = -5$ to $x = 5$ using midpoints of 5 approximating rectangles.

Sol: $\Delta x = \dfrac{b-a}{n} = \dfrac{5-(-5)}{5} = 2$ and $x_0 = -5,\ x_1 = -3,\ x_2 = -1,\ x_3 = 1,\ x_4 = 3,\ x_5 = 5.$

The mid points are $\bar{x}_1 = -4,\ \bar{x}_2 = -2,\ \bar{x}_3 = 0,\ \bar{x}_4 = 1,\ x_5 = 4$

$$M_4 = \left(f\left(\bar{x}_1\right) + f\left(\bar{x}_2\right) + f\left(\bar{x}_3\right) + f\left(\bar{x}_4\right) + f\left(\bar{x}_5\right)\right)\Delta x$$

$$= (25 - 16 + 25 - 4 + 25 - 0 + 25 - 4 + 25 - 16)\, 2 = 170$$

Q.2: If $y = \displaystyle\int_1^{\cos x} (\ln t)\, dt$, then find y''.

Sol: $y' = \ln(\cos x)\ (-\sin x)$

and $y'' = \dfrac{1}{\cos x}(-\sin x\ )(-\sin x) - \ln(\cos x)\ \cos x = \tan x \sin x - \cos x\ \ln(\cos x)$

Q.3: Using $\displaystyle\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\, \Delta x$, evaluate the integral $\displaystyle\int_0^2 \left(x^2 + 3x + 2\right) dx$

Sol: $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$ and $x_i = \dfrac{2}{n} i$

$$\int_0^2 \left(x^2 + 3x + 2\right) dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\, \Delta x = \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{4}{n^2} i^2 + \frac{3(2)}{n} i + 2\right) \frac{2}{n}$$

$$= \lim_{n\to\infty} \left(\frac{8}{n^3} \sum_{i=1}^{n} i^2 + \frac{12}{n^2} \sum_{i=1}^{n} i + \frac{4}{n} \sum_{i=1}^{n} 1\right)$$

$$= \lim_{n\to\infty} \left(\frac{8}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{12}{n^2} \frac{n(n+1)}{2} + \frac{4}{n}\ n\right)$$

$$= \lim_{n\to\infty} \left(\frac{8}{n^3} \frac{n^3\left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)}{6} + \frac{12}{n^2} \frac{n^2\left(1 + \frac{1}{n}\right)}{2} + \frac{4}{n}\ n\right)$$

$$= \frac{8}{3} + 6 + 4 = \frac{38}{3}$$