Math 102 Quiz 1 (A)

Name:..Serial# ....ID#:..............................

Q.1: Estimate area under the graph of $y = 16 - x^2$ from $x = -4$ to $x = 4$ using midpoints of 4 approximating rectangles.

Sol: $\Delta x = \dfrac{b-a}{n} = \dfrac{4-(-4)}{4} = 2$ and $x_0 = -4,\ x_1 = -2,\ x_2 = 0,\ x_3 = 2,\ x_4 = 4.$

The mid points are $\bar{x}_1 = -3,\ \bar{x}_2 = -1,\ \bar{x}_3 = 1,\ \bar{x}_4 = 3$

$$M_4 = (f(\bar{x}_1) + f(\bar{x}_2) + f(\bar{x}_3) + f(\bar{x}_4))\,\Delta x = (16-9+16-1+16-1+16-9)\,2 = 88$$

Q.2: If $y = \displaystyle\int_1^{\sin x} (\ln t)\,dt$, then find y''.

Sol: $y' = \ln(\sin x)\ \cos x$ and $y'' = \dfrac{1}{\sin x}\cos x\ \cos x - \ln(\sin x)\ \sin x = \cot x \cos x - \sin x \ln(\sin x)$

Q.3: Using $\displaystyle\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x$, evaluate the integral $\displaystyle\int_0^2 (x^3 + 2x^2 + 1)\,dx$

Sol: $\Delta x = \dfrac{2-0}{n} = \dfrac{2}{n}$ and $x_i = \dfrac{2}{n} i$

$$\int_0^2 (x^3 + 2x^2 + 1)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\,\Delta x = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{8}{n^3}i^3 + \frac{2(4)}{n^2}i^2 + 1\right)\frac{2}{n}$$

$$= \lim_{n\to\infty}\left(\frac{16}{n^4}\sum_{i=1}^{n} i^3 + \frac{16}{n^3}\sum_{i=1}^{n} i^2 + \frac{2}{n}\sum_{i=1}^{n} 1\right)$$

$$= \lim_{n\to\infty}\left(\frac{16}{n^4}\frac{n^2(n+1)^2}{4} + \frac{16}{n^3}\frac{n(n+1)(2n+1)}{6} + \frac{2}{n}\,n\right)$$

$$= \lim_{n\to\infty}\left(\frac{16}{n^4}\frac{n^4\left(1+\frac{1}{n}\right)^2}{4} + \frac{16}{n^3}\frac{n^3\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6} + \frac{2}{n}\,n\right)$$

$$= 4 + \frac{16}{3} + 2 = \frac{34}{3}.$$