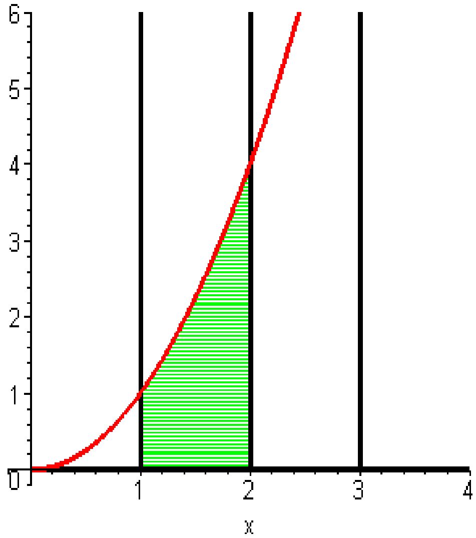
Solution Math 102 Class Test 2 (B)

Q.1: Use method of cylindrical shells to find volume of the solid generated by rotating the region bounded by the curves $y = x^2,\ y = 0,\ x = 1,\ x = 2$, about $x = 3$.

Sol:



$$V = 2\pi \int_1^2 (3 - x)\left(x^2 - 0\right) dx = 2\pi \int_1^2 \left(3x^2 - x^3\right) dx = \frac{13}{2}\pi.$$

Q.2: Find average value of the function $f(x) = \sin^3 x\ \sin 2x dx$ on the interval $\left[0, \frac{\pi}{2}\right]$.

Sol: $f_{ave} = \dfrac{1}{\frac{\pi}{2} - 0} \displaystyle\int_0^{\frac{\pi}{2}} \sin^3 x \sin 2x dx = \dfrac{1}{\frac{\pi}{2} - 0} \displaystyle\int_0^{\frac{\pi}{2}} \sin^3 x\ 2 \sin x\ \cos x\ dx = \dfrac{4}{\pi} \displaystyle\int_0^{\frac{\pi}{2}} \sin^4 x\ \cos x dx = \dfrac{4}{5\pi}.$

Q.3: Use a substitution and integration by parts to evaluate the integral $\int x^3 \cos\left(x^2\right) dx$.

Sol: Put $u = x^2$, then $du = 2xdx$

$$\int x^3 \cos\left(x^2\right) dx = \frac{1}{2} \int u \cos(u)\, du = \frac{1}{2} u \sin(u) + \frac{1}{2} \cos(u) + C = \frac{1}{2} x^2 \sin\left(x^2\right) + \frac{1}{2} \cos\left(x^2\right) + C.$$

Q.4: Evaluate the integral $\displaystyle\int \frac{3 \cos x - 5 \sin x}{\sin 2x} dx$

Sol:

$$\int \frac{3 \cos x - 5 \sin x}{\sin 2x} dx = \int \frac{3 \cos x - 5 \sin x}{2 \sin x \cos x} dx = \int \left(\frac{3 \cos x}{2 \sin x\ \cos x} - \frac{5 \sin x}{2 \sin x\ \cos x}\right) dx$$

$$= \int \left(\frac{3}{2 \sin x} - \frac{5}{2 \cos x}\right) dx = \frac{3}{2} \int \csc x dx - \frac{5}{2} \int \sec x dx$$

$$= \frac{3}{2} \ln|\csc x - \cot x| - \frac{5}{2} \ln|\sec x + \tan x| + C$$

Q.5: Evaluate the integral $\int \csc^2 x \ \cos^3(\cot x) \ dx$

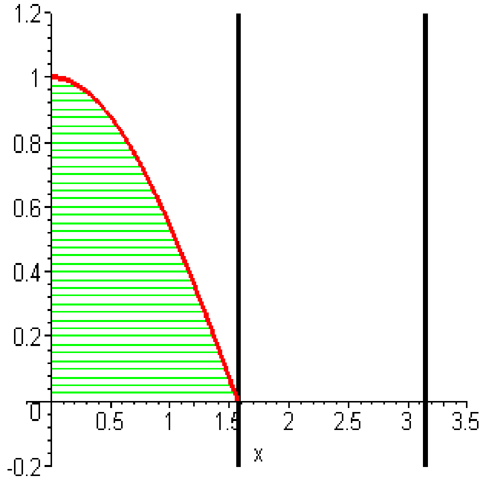
Sol: Put $u = \cot x$, then $du = -\csc^2 x dx$

$$\int \csc^2 x \ \cos^3(\cot x) \ dx = -\int \cos^3 u \, du = -\int \left(1 - \sin^2 u\right) \ \cos u \, du = -\int \cos u \, du + \int \sin^2 u \ \cos u \, du$$

$$= -\sin u + \frac{\sin^3 u}{3} + C = -\sin(\cot x) + \frac{\sin^3(\cot x)}{3} + C.$$

Q.6: Find volume of the solid generated by rotating the region bounded by the curves $y = \cos x, \ x = 0, \ x = \frac{\pi}{2}, \ y = 0$, about $x = \pi$.

Sol: Use



$$V = 2\pi \int_0^{\frac{\pi}{2}} (\pi - x)\cos x dx = 2\pi \int_0^{\frac{\pi}{2}} \pi \cos x dx - 2\pi \int_0^{\frac{\pi}{2}} x \cos x dx$$

$$= 2\pi^2 \ (\sin x)|_0^{\frac{\pi}{2}} - 2\pi \left[(x \sin x)|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x dx \right]$$

$$= 2\pi^2 - 2\pi \left(\frac{\pi}{2} - 0 + (\cos x)|_0^{\frac{\pi}{2}} \right) = 2\pi^2 - \pi^2 + 2\pi = \pi^2 + 2\pi = \pi(\pi + 2)$$