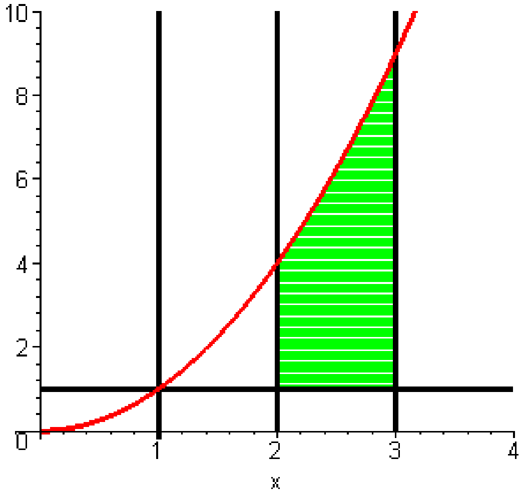
Q.1: Use method of cylindrical shells to find volume of the solid generated by rotating the region bounded by the curves $y = x^2,\ y = 1,\ x = 2,\ x = 3$, about $x = 1$.

Sol: Use



$$V = 2\pi \int_2^3 (x-1)\left(x^2-1\right)dx = 2\pi \int_2^3 \left(x^3 - x^2 - x + 1\right)dx = \frac{101}{6}\pi$$

Q.2: Find average value of the function $f(x) = \cos^3 x \sin 2x dx$ on the interval $[0, \pi]$.

Sol: $f_{ave} = \dfrac{1}{\pi - 0}\displaystyle\int_0^\pi \cos^3 x \sin 2x dx = \dfrac{1}{\pi - 0}\int_0^\pi \cos^3 x\ 2\sin x\ \cos x\ dx = \dfrac{2}{\pi}\int_0^\pi \cos^4 x\ \sin x dx = \dfrac{4}{5\pi}.$

Q.3: Use a substitution and integration by parts to evaluate the integral $\displaystyle\int x^3 e^{x^2} dx$.

Sol: Put $u = x^2$, then $du = 2xdx$

$$\int x^3 e^{x^2} dx. = \frac{1}{2}\int ue^u du = \frac{1}{2}ue^u - \frac{1}{2}e^u + C = \frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + C.$$

Q.4: Evaluate the integral $\displaystyle\int \frac{\sin x + \cos x}{\sin 2x} dx$

Sol:
$$\int \frac{\sin x + \cos x}{\sin 2x} dx = \int \frac{\sin x + \cos x}{2\sin x\ \cos x} dx = \int \left(\frac{\sin x}{2\sin x\ \cos x} + \frac{\cos x}{2\sin x\ \cos x}\right) dx$$

$$= \int \left(\frac{1}{2\cos x} + \frac{1}{2\sin x}\right) dx = \frac{1}{2}\int (\sec x + \csc x)\, dx$$

$$= \frac{1}{2}\left(\ln|\sec x + \tan x| + \ln|\csc x - \cot x|\right) + C$$

Q.5: Evaluate the integral $\int \sec^2 x \ \sin^3(\tan x) \ dx$

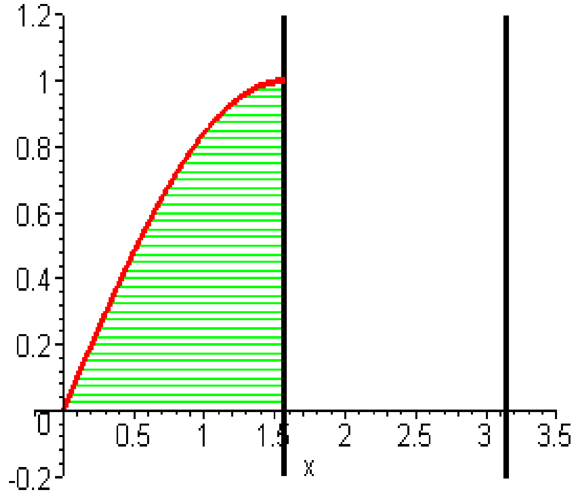
Sol: Put $u = \tan x$, then $du = \sec^2 x dx$

$$\int \sec^2 x \ \sin^3(\tan x) \ dx = \int \sin^3 u \ du = \int \left(1 - \cos^2 u\right) \ \sin u \ du = \int \sin u \ du - \int \cos^2 u \ \sin u \ du$$

$$= \cos u + \frac{\cos^3 u}{3} + C = \cos(\tan x) + \frac{\cos^3(\tan x)}{3} + C.$$

Q.6: Find volume of the solid generated by rotating the region bounded by the curves $y = \sin x, \ x = 0, \ x = \frac{\pi}{2}, \ y = 0$, about $x = \pi$.

Sol: Use



$$V = 2\pi \int_0^{\frac{\pi}{2}} (\pi - x) \sin x dx = 2\pi \int_0^{\frac{\pi}{2}} \pi \sin x dx - 2\pi \int_0^{\frac{\pi}{2}} x \sin x dx$$

$$= 2\pi^2 \left(-\cos x\right)\Big|_0^{\frac{\pi}{2}} - 2\pi \left[\left(-x \cos x\right)\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x dx \right]$$

$$= 2\pi^2 - 2\pi \left(0 + \left(\sin x\right)\Big|_0^{\frac{\pi}{2}}\right) = 2\pi^2 - 2\pi = 2\pi(\pi - 1)$$