Q.1: Evaluate the integral $\int_0^1 x^{\frac{4}{3}}\sqrt{3+x^{\frac{7}{3}}}\,dx$

Sol: Put $u = 3 + x^{\frac{7}{3}}$, then $du = \frac{7}{3}x^{\frac{4}{3}}dx$. When $x = 0,\ u = 3$ and when $x = 1,\ u = 4$.

$$\text{So} \int_0^1 x^{\frac{4}{3}}\sqrt{3+x^{\frac{7}{3}}}\,dx = \frac{3}{7}\int_3^4 \sqrt{u}du = \frac{3}{7}\frac{2}{3}\ u^{\frac{3}{2}}\Big|_3^4 = \frac{2}{7}\left(4^{\frac{3}{2}} - 3^{\frac{3}{2}}\right).$$

Q.2: Evaluate the integral $\int_0^{\pi} |\cos x|\,dx$

Sol: $$\int_0^{\pi} |\cos x|\,dx = \int_0^{\frac{\pi}{2}} \cos x\ dx + \int_{\frac{\pi}{2}}^{\pi} -\cos x\ dx = \sin x|_o^{\frac{\pi}{2}} + -\sin x|_{\frac{\pi}{2}}^{\pi} = (1-0)-(0-1) = 2.$$

Q.3: Evaluate the integral $\int_0^{\frac{3\pi}{2}} \frac{\cos^2 x}{1+\sin x}dx$

Sol: $$\int_0^{\frac{3\pi}{2}} \frac{\cos^2 x}{1+\sin x}dx = \int_0^{\frac{3\pi}{2}} \frac{1-\sin^2 x}{1+\sin x}dx = \int_0^{\frac{3\pi}{2}} \frac{(1-\sin x)(1+\sin x)}{1+\sin x}dx$$

$$= \int_0^{\frac{3\pi}{2}} (1-\sin x)\,dx = (x+\cos x)|_0^{\frac{3\pi}{2}} = \tfrac{3\pi}{2} - 0 - 0 - 1 = \tfrac{3\pi}{2} - 1$$

Q.4: Evaluate the integral $\int \frac{\csc x\ \cot x}{1+\csc^2 x}dx$

Sol: Put $u = \csc x$, then $du = -\csc x\ \cot x dx$

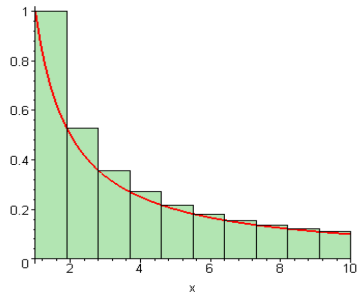
$$\int \frac{\csc x\ \cot x}{1+\csc^2 x}dx = \int \frac{-1}{1+u^2}du = \cot^{-1} u + C = \cot^{-1}(\csc x) + C$$

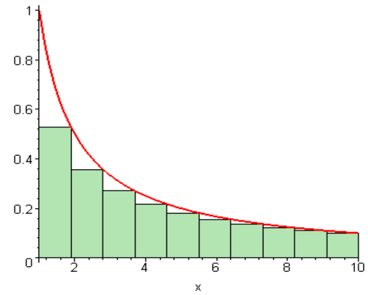
Q.5: By comparing areas, show that $\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < \ln n < 1 + \frac{1}{2} + \cdots + \frac{1}{n-1}$. (Show all your work)

Sol: $\ln n = \int_1^n \frac{1}{t}dt$

The area under the graph of $f(t) = \frac{1}{t}$ from $t = 1$ to $t = n$ is $\int_1^n \frac{1}{t}dt$.

$\int_1^n \frac{1}{t}dt$ can be approximated by $n-1$ rectangles $L_{n-1} = 1 + \frac{1}{2} + \cdots + \frac{1}{n-1}$ which is an over estimate



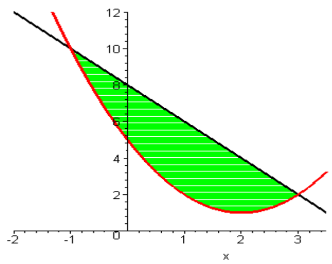


and also by $R_{n-1} = \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ which is an under estimate.of

So $\frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} < \ln n < 1 + \frac{1}{2} + \cdots + \frac{1}{n-1}$

Q.6: Find area of the region bounded by the curves $y = x^2 - 4x + 5$ and $y = -2x + 8$.

Sol:

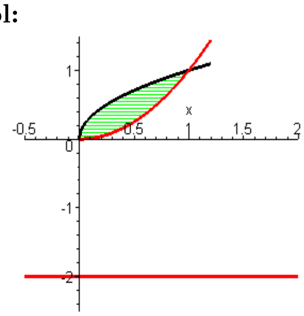


$$x^2 - 4x + 5 = -2x + 8 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x+1)(x-3) = 0 \Rightarrow x = -1, 3.$$

$$A = \int_{-1}^{3} \left(-2x + 8 - x^2 + 4x - 5\right) dx = \int_{-1}^{3} \left(2x + 3 - x^2\right) dx = \left(x^2 + 3x - \frac{x^3}{3}\right)\Big|_{-1}^{3} = \frac{32}{3}$$

Q.7: Find volume of solid obtained by rotating the region bounded by $y = x^2$, and $x = y^2$ about the line $y = -2$.

Sol:



$$A(y) = \pi\left[\left(\sqrt{x} + 2\right)^2 - \left(x^2 + 2\right)^2\right] = \pi\left[x + 4\sqrt{x} + 4 - x^4 - 4x^2 - 4\right] = \pi\left[x + 4\sqrt{x} - x^4 - 4x^2\right]$$

$$V = \pi \int_{0}^{1} \left(x + 4\sqrt{x} - x^4 - 4x^2\right) dx = \frac{49}{30}\pi$$