Solution Math 101   Quiz 2   (B)

Q.1: Find the numbers c and d such that the function $f(x) = \left\{ \begin{array}{cc} 3cx - 4, & \text{if } x < 1 \\ 2x^2 - 5cx + d, & \text{if } 1 \leq x < 2 \\ d(2x-1) - 2, & \text{if } x \geq 2 \end{array} \right\}$ is continuous.

Sol: $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} (3cx - 4) = 3c - 4$ and $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} (2x^2 - 5cx + d) = 2 - 5c + d$

Continuity at $x = 1$ implies $2 - 5c + d = 3c - 4$ or $8c - d = 6$.

$\lim\limits_{x\to 2^-} (2x^2 - 5cx + d) = 8 - 10c + d$ and $\lim\limits_{x\to 2^+} d(2x-1) - 2 = 3d - 2$

Continuity at $x = 2$ implies $8 - 10c + d = 3d - 2$ or $10c + 2d = 10$

Solving $8c - d = 6$ and $10c - 2d = 10$, we get $c = \dfrac{11}{13}$ and $d = \dfrac{10}{13}$.

Q.2: Determine the limit $\lim\limits_{x\to\infty} \dfrac{(3x^2+2)^2}{(2x-3)(3x^3+1)}$

$$\lim_{x\to\infty} \frac{(3x^2+2)^2}{(2x-3)(3x^3+1)} = \lim_{x\to\infty} \frac{9x^4 + 12x^2 + 4}{6x^4 - 9x^3 + 2x - 3} = \frac{9}{6} = \frac{3}{2}.$$

Q.3: Find horizontal and vertical asymptotes of the function $y = \dfrac{(3x^2+2)^2}{(2x-3)(3x^3+1)}$.

Sol: Horizontal Asymptote: $y = \lim\limits_{x\to\infty} \dfrac{(3x^2+2)^2}{(2x-3)(3x^3+1)} = \dfrac{3}{2}$.

Vertical Asymptotes: $x = a$ is a vertical asymptote if a is a zero of the denominator.

Zeros of the denominator are $\dfrac{3}{2}$ and $-\left(\dfrac{1}{3}\right)^{\frac{1}{3}}$.

So vertical asymptotes are $x = \dfrac{3}{2}$ and $x = -\left(\dfrac{1}{3}\right)^{\frac{1}{3}}$.